

INVESTIGATIONS ON \mathcal{T} VIOLATION AND \mathcal{CPT} SYMMETRY IN THE NEUTRAL KAON SYSTEM

– a pedagogical choice –

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Abstract

During the recent years experiments with neutral kaons have yielded remarkably sensitive results which are pertinent to such fundamental phenomena as \mathcal{CPT} invariance (protecting causality), time-reversal invariance violation, coherence of wave functions, and entanglement of kaons in pair states. We describe the phenomenological developments and the theoretical conclusions drawn from the experimental material. An outlook to future experimentation is indicated.

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1 Introduction

A neutral kaon oscillates forth and back between itself and its antiparticle, because the physical quantity, strangeness, which distinguishes antikaons from kaons, is not conserved by the interaction which governs the time evolution [1, 2].

The observed behaviour is the interference pattern of the matter waves of the K_S and of the K_L , which act as highly stable damped oscillators with a relative frequency difference of $\Delta m/m_{K^0} \approx 7 \times 10^{-15}$. The corresponding beat frequency of $\omega = \Delta m/\hbar \approx 5.3 \times 10^9 \text{ s}^{-1}$, and the scale of the resulting spatial interference pattern of $2\pi c/\omega \approx 0.36 \text{ m}$, fit well the present technical capabilities of detectors in high-energy physics.

Some fraction of the size of $\Delta m \approx 3.5 \times 10^{-15} \text{ GeV}$, depending on the measurement's precision, indicates the magnitude of effects, which may be present inside of the equation of motion, to which the measurements are sensitive: 10^{-18} to 10^{-21} GeV .

This article describes the relations of the parameters, which specify the properties of the laws of the time evolution and of the decays of the neutral kaons, to the measured quantities, with emphasis to CPT invariance and to T violations.

Experimental tests of CPT invariance are of great interest. This invariance plays a fundamental rôle for a causal description of physical phenomena. We quote Res Jost [3], pioneer of the CPT theorem [4–8], '... : eine kausale Beschreibung ist nur möglich, wenn man den Naturgesetzen eine Symmetrie zugesteht, welche den Zeitpfeil umkehrt'. We shall describe experimental results that could have contradicted CPT invariance, but did not.

T violation accompanies CP violation [9]. Recently, an experiment [10] at CERN has measured that the evolution $\bar{K}^0 \rightarrow K^0$ is faster than $K^0 \rightarrow \bar{K}^0$, which, as explained below, formally contradicts T invariance in the neutral kaon's weak interaction.

T and CPT violation have CP violation as a necessary condition. For this one, a simple criterion on the quark level has been given in the form of one single relation [11].

We also consider T violation versus T oddness.

We shall discuss the general time evolution as well of a single neutral kaon as of an entangled pair of neutral kaons by applying the density matrix formalism. This allows one to design experiments which test, whether isolated kaons in a pure state do not evolve into ones in a mixed state. Such transitions would be forbidden by quantum mechanics and would violate CPT invariance.

First experimental results on this subject have been obtained at CERN [12].

In our description we shall, as most authors do, assume, that the kaon's time evolution, and the decay, are distinct processes. The time evolution is derived from a Schrödinger equation, and it becomes parametrized via perturbation theory ($\rightarrow \Lambda_{\alpha\alpha'}$), while specific decays are described by time-independent amplitudes. The conservation of probability, inherent in the perturbation result, links the two processes.

We present the results which concern CPT invariance and T violation derived mostly from neutral kaon decays into $\pi\pi$ and $e\pi\nu$ final states.

The history of the unveilings of the characteristics of the neutral kaons is a sparkling succession of brilliant ideas and achievements, theoretical and experimental [13–19].

We have reasons to assume that neutral kaons will enable one to make more basic discoveries. Some of the kaons' properties (e. g. entanglement) have up to now only scarcely been exploited.

2 The neutral-kaon system

2.1 Time evolution

The time evolution of a neutral kaon and of its decay products may be represented by the state vector

$$|\psi\rangle = \psi_{K^0}(t) |K^0\rangle + \psi_{\bar{K}^0}(t) |\bar{K}^0\rangle + \sum_m c_m(t) |m\rangle \quad (1)$$

which satisfies the Schrödinger equation

$$i \frac{d|\psi\rangle}{dt} = \mathcal{H} |\psi\rangle. \quad (2)$$

In the Hamiltonian, $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{wk}}$, \mathcal{H}_0 governs the strong and electromagnetic interactions. It is invariant with respect to the transformations \mathcal{C} , \mathcal{P} , \mathcal{T} , and it conserves the strangeness S . The states $|K^0\rangle$ and $|\bar{K}^0\rangle$ are common stationary eigenstates of \mathcal{H}_0 and S , with the mass m_0 and with opposite strangeness: $\mathcal{H}_0 |K^0\rangle = m_0 |K^0\rangle$, $\mathcal{H}_0 |\bar{K}^0\rangle = m_0 |\bar{K}^0\rangle$, $S |K^0\rangle = |K^0\rangle$, $S |\bar{K}^0\rangle = -|\bar{K}^0\rangle$. The states $|m\rangle$ are continuum eigenstates of \mathcal{H}_0 and represent the decay products. They are absent at the instant of production ($t = 0$) of the neutral kaon. The initial condition is thus

$$|\psi_0\rangle = \psi_{K^0}(0) |K^0\rangle + \psi_{\bar{K}^0}(0) |\bar{K}^0\rangle . \quad (3)$$

\mathcal{H}_{wk} governs the weak interactions. Since these do not conserve strangeness, a neutral kaon will, in general, change its strangeness as time evolves.

Equation (2) may be solved for the unknown functions $\psi_{K^0}(t)$ and $\psi_{\bar{K}^0}(t)$, by using a perturbation approximation [20] which yields [21, 22]

$$\psi = e^{-i\Lambda t} \psi_0 . \quad (4)$$

ψ is the column vector with components $\psi_{K^0}(t)$ and $\psi_{\bar{K}^0}(t)$, ψ_0 equals ψ at $t = 0$, and Λ is the time-independent 2×2 matrix ($\Lambda_{\alpha\alpha'}$), whose components refer to the two-dimensional basis $|K^0\rangle$, $|\bar{K}^0\rangle$ and may be written as $\Lambda_{\alpha\alpha'} = \langle\alpha|\Lambda|\alpha'\rangle$ with $\alpha, \alpha' = K^0, \bar{K}^0$.

Since the kaons decay, we have

$$0 > \frac{d|\psi|^2}{dt} = -i\psi^\dagger (\Lambda - \Lambda^\dagger) \psi . \quad (5)$$

Λ is thus not hermitian, $e^{-i\Lambda t}$ is not unitary, in general.

This motivates the definition of M and Γ as

$$\Lambda = M - \frac{i}{2}\Gamma , \quad (6a)$$

$$M = M^\dagger, \quad \Gamma = \Gamma^\dagger . \quad (6b)$$

We find

$$0 > \frac{d|\psi|^2}{dt} = -\psi^\dagger \Gamma \psi . \quad (7)$$

This expresses that Γ has to be a positive matrix.

The perturbation approximation also establishes the relation from \mathcal{H}_{wk} to Λ (including second order in \mathcal{H}_{wk}) by

$$M_{\alpha\alpha'} = m_0 \delta_{\alpha\alpha'} + \langle\alpha|\mathcal{H}_{\text{wk}}|\alpha'\rangle + \mathcal{P} \sum_{\beta} \left(\frac{\langle\alpha|\mathcal{H}_{\text{wk}}|\beta\rangle \langle\beta|\mathcal{H}_{\text{wk}}|\alpha'\rangle}{m_0 - E_{\beta}} \right) , \quad (8a)$$

$$\Gamma_{\alpha\alpha'} = 2\pi \sum_{\beta} \langle\alpha|\mathcal{H}_{\text{wk}}|\beta\rangle \langle\beta|\mathcal{H}_{\text{wk}}|\alpha'\rangle \delta(m_0 - E_{\beta}) , \quad (8b)$$

$$(\alpha, \alpha' = K^0, \bar{K}^0) .$$

Equations (8a, 8b) enable one now to state directly the symmetry properties of \mathcal{H}_{wk} in terms of experimentally observable relations among the elements of Λ , see Table 1. We remark that \mathcal{CPT} invariance imposes no restrictions on the off-diagonal elements, and that \mathcal{T} invariance imposes no restrictions on the diagonal elements of Λ . \mathcal{CP} invariance is violated, whenever one, at least, of these invariances is

Table 1: The symmetry properties of \mathcal{H}_{wk} induce symmetries in observable quantities.

The last column indicates asymmetries of quantities which have been measured by the CPLEAR experiment [23] at CERN. More explanations are given in the text.

| If \mathcal{H}_{wk} has the property | called | then | or |
|--|----------------------------|--|----------------------|
| $\mathcal{T}^{-1} \mathcal{H}_{\text{wk}} \mathcal{T} = \mathcal{H}_{\text{wk}}$ | \mathcal{T} invariance | $ \Lambda_{K^0 \bar{K}^0} = \Lambda_{\bar{K}^0 K^0} $ | $A_T = 0$ |
| $(\mathcal{CPT})^{-1} \mathcal{H}_{\text{wk}} (\mathcal{CPT}) = \mathcal{H}_{\text{wk}}$ | \mathcal{CPT} invariance | $\Lambda_{K^0 K^0} = \Lambda_{\bar{K}^0 \bar{K}^0}$ | $A_{\text{CPT}} = 0$ |
| $(\mathcal{CP})^{-1} \mathcal{H}_{\text{wk}} (\mathcal{CP}) = \mathcal{H}_{\text{wk}}$ | \mathcal{CP} invariance | $\Lambda_{K^0 K^0} = \Lambda_{\bar{K}^0 \bar{K}^0}$ and $ \Lambda_{K^0 \bar{K}^0} = \Lambda_{\bar{K}^0 K^0} $ | $A_{\text{CP}} = 0$ |

violated.

The definitions of $|K^0\rangle$ and $|\bar{K}^0\rangle$ leave a real phase ϑ undetermined:

Since $S^{-1} \mathcal{H}_0 S = \mathcal{H}_0$, the states

$$|K^{0'}\rangle = e^{i\vartheta S} |K^0\rangle = e^{i\vartheta} |K^0\rangle, \quad (9a)$$

$$|\bar{K}^{0'}\rangle = e^{i\vartheta S} |\bar{K}^0\rangle = e^{-i\vartheta} |\bar{K}^0\rangle. \quad (9b)$$

fulfil the definitions of $|K^0\rangle$ and $|\bar{K}^0\rangle$ as well, and constitute thus an equivalent basis which is related to the original basis by a unitary transformation. As the observables are always invariant with respect to unitary base transformations, the parameter ϑ cannot be measured, and remains undetermined. This has the effect that expressions which depend on ϑ are not suited to represent experimental results, unless ϑ has beforehand been fixed to a definite value by convention. Although such a convention may simplify sometimes the arithmetic, it risks to obscure the insight as to whether a certain result is genuine or whether it is just an artifact of the convention.

As an example we consider the elements of Λ , $\Lambda_{\alpha\alpha'} = \langle\alpha|\Lambda|\alpha'\rangle$ which refer to the basis $\alpha, \alpha' = K^0, \bar{K}^0$. With respect to the basis $e^{i\vartheta} K^0, e^{-i\vartheta} \bar{K}^0$ we obtain the same diagonal elements, whereas the off-diagonal elements change into

$$\Lambda_{K^0 \bar{K}^0} \mapsto \Lambda'_{K^0 \bar{K}^0} = e^{-2i\vartheta} \Lambda_{K^0 \bar{K}^0}, \quad (10a)$$

$$\Lambda_{\bar{K}^0 K^0} \mapsto \Lambda'_{\bar{K}^0 K^0} = e^{2i\vartheta} \Lambda_{\bar{K}^0 K^0}, \quad (10b)$$

and are thus convention dependent. However, their product, their absolute values, the trace $\text{tr}\{\Lambda\}$, its determinant, and its eigenvalues (not so its eigenvectors), but also the partition into M and Γ , are convention independent [24]. (We will introduce a phase convention later in view of comparing experimental results).

The definition of the operations \mathcal{CP} and \mathcal{CPT} allows one to define two additional phase angles. We select them such that $\mathcal{O}K^0 = \bar{K}^0$ and $\mathcal{O}\bar{K}^0 = K^0$, where \mathcal{O} stands for \mathcal{CP} or \mathcal{CPT} . See e. g. [25].

In order to describe the time evolution of a neutral kaon, the matrix exponential $e^{-i\Lambda t}$ has to be calculated.

If the exponent matrix has *two* dimensions, a generalized Euler formula gives a straightforward answer. Be Λ represented as a superposition of Pauli matrices

$$\Lambda = \Lambda^\mu \sigma^\mu \quad (11)$$

with $\sigma^0 =$ unit matrix, $\sigma^k =$ Pauli matrices, Λ^μ complex. (Summation over multiple indices: Greek 0 to 3, Roman 1 to 3).

Then

$$e^{-i\Lambda t} = e^{-i\Lambda^\mu \sigma^\mu t} = e^{-i\Lambda^0 t} (\sigma^0 \cos(\Omega t) - i\Lambda^m \sigma^m t \sin(\Omega t)/(\Omega t)), \quad (12)$$

where $\Omega = (\Lambda^m \Lambda^m)^{1/2}$. Noting that $\Lambda^\mu = \frac{1}{2} \text{tr}\{\sigma^\mu \Lambda\}$, we see that Eq. (12) expresses $e^{-i\Lambda t}$ entirely in terms of the elements of Λ . Since the (complex) eigenvalues λ_L, λ_S of Λ turn out to be observable (and are thus doubtlessly phase transformation invariant) we introduce them into (12).

They fulfil $\lambda_L \lambda_S = \det(\Lambda) = \Lambda^0 \Lambda^0 - \Lambda^m \Lambda^m$, and $\lambda_L + \lambda_S = \text{tr}\{\Lambda\} = 2\Lambda^0$, and thus, with $\Delta\lambda \equiv \lambda_L - \lambda_S$,

$$\Omega = \Delta\lambda/2. \quad (13)$$

We note here (with relief) that the calculation of the general time evolution, expressed in Eq. (4), does not need the knowledge of the *eigenstates* of Λ , whose physical interpretation needs special attention [26, 27].

The following corollary will be of interest:

The off-diagonal elements of a 2×2 exponent matrix factorize the off-diagonal elements of its matrix exponential, with equal factors:

$$(e^{-i\Lambda t})_{j \neq k} = (-i\Lambda t)_{j \neq k} e^{-\frac{1}{2}i(\Lambda_{K^0 K^0} + \Lambda_{\bar{K}^0 \bar{K}^0})t} \sin(\Omega t)/(\Omega t). \quad (14)$$

This is valid for two dimensions.

Independent of the dimension n of the exponent matrix, diagonalization allows one to calculate the matrix exponential. Find the two vectors $|K_{L,S}\rangle$ which Λ transforms into a multiple of themselves

$$\Lambda |K_{L,S}\rangle = \lambda_{L,S} |K_{L,S}\rangle. \quad (15)$$

The eigenvalues $\lambda_{L,S}$ need to be

$$\lambda_{L,S} = \frac{1}{2} \text{tr}\{\Lambda\} \pm \sqrt{(\text{tr}\{\Lambda\})^2/4 - \det(\Lambda)}. \quad (16)$$

We may express the solutions of (15) in the basis $|K^0\rangle, |\bar{K}^0\rangle$ as

$$|K_S\rangle = V^{11} |K^0\rangle + V^{21} |\bar{K}^0\rangle \doteq \begin{pmatrix} V^{11} \\ V^{21} \end{pmatrix} \quad (17)$$

$$|K_L\rangle = V^{12} |K^0\rangle + V^{22} |\bar{K}^0\rangle \doteq \begin{pmatrix} V^{12} \\ V^{22} \end{pmatrix} \quad (18)$$

and form the matrix $V = (V^{ij})$ whose columns are the components of the eigenvectors, and also $W = V^{-1}$. The matrix Λ can now be represented as

$$\Lambda = VDW \quad (19)$$

where D is diagonal,

$$D = \begin{pmatrix} \lambda_S & 0 \\ 0 & \lambda_L \end{pmatrix}.$$

Since we need to extract V and W from the exponent to obtain

$$e^{-i\Lambda t} = e^{-iVDWt} = V e^{-iDt} W, \quad (20)$$

it is important that $W = V^{-1}$ (and *not* $W = V^\dagger \neq V^{-1}$). Since $WV = 1$ or $W^{ij}V^{jk} = \delta^{ik}$, the rows of W are orthogonal to the columns of V . A convenient solution is

$$W = (W^{ij}) = \frac{1}{|V|} \begin{pmatrix} V^{22} & -V^{12} \\ -V^{21} & V^{11} \end{pmatrix}. \quad (21)$$

Inserting V , D , W into (20) allows one to express $e^{-i\Lambda t}$ in terms of the eigenelements $|K_{L,S}\rangle$, and $\lambda_{L,S}$. (Eq. (19) also shows how to construct a matrix with prescribed (non-parallel) eigenvectors and independently prescribed eigenvalues).

If we define the vectors

$$\langle \widetilde{K}_S | = W^{11} \langle K^0 | + W^{12} \langle \overline{K}^0 | \quad (22)$$

$$\langle \widetilde{K}_L | = W^{21} \langle K^0 | + W^{22} \langle \overline{K}^0 | \quad (23)$$

then we have

$$\langle \widetilde{K}_\kappa | K_{\kappa'} \rangle = \delta_{\kappa\kappa'}, \quad \kappa, \kappa' = L, S \quad (24)$$

in contrast to

$$\langle K_\kappa | K_{\kappa' \neq \kappa} \rangle \neq 0. \quad (25)$$

The difficulty to interpret these vectors as states is discussed in [26]. Eq. (25) shows that there is no clear state K_L , because the vector $|K_L\rangle$ always has a component of $|K_S\rangle$, with the probability $|\langle K_S | K_L \rangle|^2$.

We now solve (15): $\Lambda^{ij}V^{j\kappa} = \lambda_\kappa V^{i\kappa}$, (no sum κ) for $V^{j\kappa} \triangleq |K_\kappa\rangle$, $\kappa = (1, 2) \triangleq (S, L)$, and regain (12) in a different form:

$$e^{-i\Lambda t} = \begin{pmatrix} f_+ + 2\delta f_- & -2\Lambda_{K^0\overline{K}^0}f_-/\Delta\lambda \\ -2\Lambda_{\overline{K}^0K^0}f_-/\Delta\lambda & f_+ - 2\delta f_- \end{pmatrix} = \begin{pmatrix} f_+ + 2\delta f_- & -2(\sigma - \varepsilon)f_- \\ -2(\sigma + \varepsilon)f_- & f_+ - 2\delta f_- \end{pmatrix} \quad (26)$$

with

$$f_\pm(t) = \frac{e^{-i\lambda_S t} \pm e^{-i\lambda_L t}}{2}, \quad (27)$$

$$\delta \equiv (\Lambda_{\overline{K}^0\overline{K}^0} - \Lambda_{K^0K^0})/(2\Delta\lambda), \quad (28a)$$

$$\varepsilon \equiv (\Lambda_{\overline{K}^0K^0} - \Lambda_{K^0\overline{K}^0})/(2\Delta\lambda), \quad (28b)$$

$$\sigma \equiv (\Lambda_{\overline{K}^0K^0} + \Lambda_{K^0\overline{K}^0})/(2\Delta\lambda). \quad (28c)$$

We have set

$$\lambda_{L,S} = m_{L,S} - \frac{i}{2}\Gamma_{L,S} \quad (29)$$

and

$$\Delta\lambda = \lambda_L - \lambda_S = m_L - m_S + \frac{i}{2}(\Gamma_S - \Gamma_L) \equiv \Delta m + \frac{i}{2}\Delta\Gamma = |\Delta\lambda| e^{i(\frac{\pi}{2} - \phi_{SW})} \quad (30)$$

with

$$\Delta m \equiv m_L - m_S, \quad \Delta\Gamma \equiv \Gamma_S - \Gamma_L, \quad (31)$$

and with ϕ_{SW} defined by

$$\tan(\phi_{SW}) = (2\Delta m / \Delta\Gamma). \quad (32)$$

The parameters in (28) satisfy the identity

$$\sigma^2 - \varepsilon^2 + \delta^2 \equiv 1/4 \quad (33)$$

which entails

$$\zeta \equiv |\sigma|^2 + |\varepsilon|^2 + |\delta|^2 - 1/4 \geq 0. \quad (34)$$

From Table 1 we can deduce that ζ signifies the violations of \mathcal{T} and $\mathcal{CP}\mathcal{T}$.

The positivity of the matrix Γ requires the determinant $|\Gamma|$ to be positive

$$0 < |\Gamma| = |\Delta\lambda|^2 \left(\frac{\Gamma_S \Gamma_L}{|\Delta\lambda|^2} - 2\zeta \right), \quad (35)$$

which needs

$$\zeta < \frac{\Gamma_S \Gamma_L}{2|\Delta\lambda|^2} \approx \frac{\Gamma_L}{\Gamma_S}. \quad (36)$$

The last approximation is valid for neutral kaons where, experimentally, $2|\Delta\lambda|^2 \approx (\Gamma_S)^2$.

We see from (36), that the ratio Γ_L/Γ_S ($\approx 1.7 \times 10^{-3}$) provides a general limit for the violations of \mathcal{T} and $\mathcal{CP}\mathcal{T}$ invariance.

The eigenstates can now be expressed by the elements of Λ

$$|K_S\rangle = N_S (\Lambda_{K^0 \bar{K}^0} |K^0\rangle + (\lambda_S - \Lambda_{K^0 K^0}) |\bar{K}^0\rangle), \quad (37)$$

$$|K_L\rangle = N_L ((\Lambda_{\bar{K}^0 \bar{K}^0} - \lambda_L) |K^0\rangle - \Lambda_{\bar{K}^0 K^0} |\bar{K}^0\rangle), \quad (38)$$

with suitable normalization factors N_S, N_L .

They develop in time according to

$$|K_{L,S}\rangle \rightarrow e^{-i\lambda_{L,S}t} |K_{L,S}\rangle.$$

$\Gamma_{L,S}$ thus signify the decay widths of the eigenstates with mean lifes $\tau_{L,S} = 1/\Gamma_{L,S}$, and $m_{L,S}$ are the rest masses.

These quantities are directly measurable. The results show that $\tau_L \gg \tau_S$ and $m_L > m_S$. We therefore have

$$0 \leq \phi_{SW} \leq \pi/2.$$

In the limit $\varepsilon \rightarrow 0, \delta \rightarrow 0$, the eigenstates are

$$|K_S\rangle \rightarrow |K_1\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle),$$

$$|K_L\rangle \rightarrow |K_2\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle).$$

The differences $M_{K^0 K^0} - M_{\bar{K}^0 \bar{K}^0}$ and $\Gamma_{K^0 K^0} - \Gamma_{\bar{K}^0 \bar{K}^0}$, which may be interpreted as ($\mathcal{CP}\mathcal{T}$ violating) mass and decay width differences between the K^0 and the \bar{K}^0 , are related to δ and to $\Delta\lambda$ as follows:

Define the reals δ_{\parallel} and δ_{\perp} by

$$\delta_{\parallel} + i\delta_{\perp} = \delta e^{-i\phi_{SW}} \quad (39)$$

then

$$M_{K^0 K^0} - M_{\bar{K}^0 \bar{K}^0} = 2|\Delta\lambda| \delta_{\perp} \quad (40)$$

$$\Gamma_{K^0 K^0} - \Gamma_{\bar{K}^0 \bar{K}^0} = 4|\Delta\lambda| \delta_{\parallel}. \quad (41)$$

We wish to remark that, given the constants $|\Delta\lambda|$ and ϕ_{SW} , the information contained in the mass and decay width differences (40,41) is identical to the one in δ .

2.2 Symmetry

The measurement of particularly chosen transition rate asymmetries concerning the neutral kaon's time evolution exploit properties of \mathcal{H}_{wk} in an astonishingly direct way.

To explain the principle of the choice of the observables we make the temporary assumption that the detected decay products unambiguously mark a relevant property of the kaon at the moment of its decay: The decay into two pions indicates a \mathcal{CP} eigenstate with a positive eigenvalue, a semileptonic decay, ($\rightarrow e\pi\nu$ or $\rightarrow \mu\pi\nu$), indicates the kaon's strangeness to be equal to the charge of the lepton (in units of positron charge).

We will later show that previously unknown symmetry properties of the decay mechanism ($\Delta S = \Delta Q$ rule, \mathcal{CPT} violation 'in decay') or practical experimental conditions (efficiencies, interactions with the detector material, regeneration of K_S by matter) do not change the conclusions of this section.

2.2.1 \mathcal{T} violation

Compare the probability for an antikaon to develop into a kaon, $|\langle K^0 | e^{-i\Lambda t} | \bar{K}^0 \rangle|^2$, with the one for a kaon to develop into an antikaon, $|\langle \bar{K}^0 | e^{-i\Lambda t} | K^0 \rangle|^2$, within the same time interval t . Intuition wants the probabilities for these mutually reverse processes to be the same, if time reversal invariance holds.

We now show that the experimentally observed difference [10] formally contradicts \mathcal{T} invariance in \mathcal{H}_{wk} .

Following [28], time reversal invariance, defined by $\mathcal{T}^{-1} \mathcal{H}_{\text{wk}} \mathcal{T} = \mathcal{H}_{\text{wk}}$, requires

$$\Gamma_{\bar{K}^0 K^0}^* / \Gamma_{\bar{K}^0 \bar{K}^0} = M_{\bar{K}^0 K^0}^* / M_{\bar{K}^0 \bar{K}^0} \quad (42)$$

which is equivalent to $|\Lambda_{K^0 \bar{K}^0}|^2 = |\Lambda_{\bar{K}^0 K^0}|^2$. This is measurable !
The normalized difference of these quantities

$$A_T \equiv \frac{|\Lambda_{K^0 \bar{K}^0}|^2 - |\Lambda_{\bar{K}^0 K^0}|^2}{|\Lambda_{K^0 \bar{K}^0}|^2 + |\Lambda_{\bar{K}^0 K^0}|^2} \quad (43)$$

is a theoretical measure for time reversal violation [29], and we find, using (14), the identity

$$A_T \equiv \frac{|\langle K^0 | e^{-i\Lambda t} | \bar{K}^0 \rangle|^2 - |\langle \bar{K}^0 | e^{-i\Lambda t} | K^0 \rangle|^2}{|\langle K^0 | e^{-i\Lambda t} | \bar{K}^0 \rangle|^2 + |\langle \bar{K}^0 | e^{-i\Lambda t} | K^0 \rangle|^2}, \quad (44)$$

which expresses the different transition probabilities for the mutually reverse processes $\bar{K}^0 \longleftrightarrow K^0$, as a formal consequence of the property of \mathcal{H}_{wk} not to commute with \mathcal{T} .

The value of A_T is predicted as follows

$$A_T = \frac{-2\text{Re}(\varepsilon\sigma^*)}{|\varepsilon|^2 + |\sigma|^2} \approx 4\text{Re}(\varepsilon) \quad (\text{for } |\varepsilon| \ll |\sigma| \text{ and } \sigma \approx -\frac{1}{2}). \quad (45)$$

We add some general remarks:

The directness of the relation between A_T and \mathcal{H}_{wk} rests partly on the fact that the neutral kaons are described in a two dimensional space, (K^0, \bar{K}^0) , in which the corollary (14) is valid. This is also the origin for the time independence of A_T [30].

A_T is a \mathcal{T} -odd quantity insofar as it changes its sign under the interchange of the initial and final states $\bar{K}^0 \longleftrightarrow K^0$.

Eqs. (42) and (43) describe time reversal invariance in an explicitly phase transformation invariant form. In Eq. (45), both, the numerator and the denominator, have this invariance. The approximations concerning $|\varepsilon|$ and $|\sigma|$ correspond to the phase convention to be introduced later. (We will choose a phase angle

ϑ , neglecting $|\varepsilon|^2 \ll 1$ and $|\delta|^2 \ll 1$, such that $\sigma = -1/2$).

Eq. (42) shows that the present two dimensional system can manifest time reversal violation only, if Γ is not the null matrix, i. e. if there is decay. However, since the absolute value $|\Gamma_{\bar{K}^0 K^0}|$ does not enter Eq. (42), the definition of time reversal invariance would stay intact if the decay rates Γ_S and Γ_L would (hypothetically) become the same, contradicting [31, 32].

A_T has been measured [10] not to vanish, $A_T \neq 0$. Since only the relative phase of $\Gamma_{\bar{K}^0 K^0}$ and $M_{\bar{K}^0 K^0}$, $\arg(\Gamma_{\bar{K}^0 K^0}) - \arg(M_{\bar{K}^0 K^0})$, and not the absolute values, determines time reversal violation, Eq. (42) does not give any prescription as to what extent the violation should be attributed to M or to Γ .

2.2.2 \mathcal{CPT} invariance

The \mathcal{CPT} invariance of \mathcal{H}_{wk} requires the equality of the probabilities, for a kaon and for an antikaon, to develop into themselves.

$$A_{\text{CPT}} \equiv \frac{|\langle \bar{K}^0 | e^{-i\Lambda t} | \bar{K}^0 \rangle|^2 - |\langle K^0 | e^{-i\Lambda t} | K^0 \rangle|^2}{|\langle \bar{K}^0 | e^{-i\Lambda t} | \bar{K}^0 \rangle|^2 + |\langle K^0 | e^{-i\Lambda t} | K^0 \rangle|^2} \quad (46)$$

is thus a measure for a possible \mathcal{CPT} violation. We note (from Ref. [28]), indicated in Table 1, that \mathcal{CPT} invariance entails $\Lambda_{K^0 \bar{K}^0} = \Lambda_{\bar{K}^0 K^0}$, or $\delta = 0$. Using (12), we obtain, with $|\delta| \ll 1$,

$$A_{\text{CPT}} = \frac{4\text{Re}(\delta) \sinh(\frac{1}{2}\Delta\Gamma t) + 4\text{Im}(\delta) \sin(\Delta m t)}{\cosh(\frac{1}{2}\Delta\Gamma t) + \cos(\Delta m t)}, \quad (47)$$

and confirm that $A_{\text{CPT}} \neq 0$ at any time, i. e. $\delta \neq 0$, would contradict the property of \mathcal{H}_{wk} to commute with \mathcal{CPT} .

2.3 Decays

We assume that the creation, the evolution, and the decay of a neutral kaon can be considered as a succession of three distinct and independent processes. Each step has its own amplitude with its particular properties. It determines the initial conditions for the succeeding one. (For a refined treatment which considers the kaon as a virtual particle, see [33, 34], with a comment in [26]).

The amplitude for a kaon characterized by $|\psi_0\rangle$ at $t = 0$, decaying at time t , into a state $|\psi_f\rangle$ is given by

$$\begin{aligned} \mathcal{A}^f &= \langle \psi_f | \mathcal{H}_{\text{wk}} | \psi \rangle = \langle \psi_f | \mathcal{H}_{\text{wk}} e^{-i\Lambda t} | \psi_0 \rangle = \langle \psi_f | \mathcal{H}_{\text{wk}} | s' \rangle \langle s' | e^{-i\Lambda t} | s \rangle \langle s | \psi_0 \rangle \\ &= \mathcal{A}_{s's}^f (e^{-i\Lambda t})_{s's} \psi_s(0). \end{aligned} \quad (48)$$

The sum over s', s includes all existent, unobservable, (interfering) paths.

It is

$$\mathcal{A}_{s's}^f = \langle \psi_f | \mathcal{H}_{\text{wk}} | s' \rangle \quad (49)$$

the amplitude for the instantaneous decay of the state with strangeness s' into the state $|\psi_f\rangle$, and $\psi_s(0) = \langle s | \psi_0 \rangle$, ($s = K^0, \bar{K}^0$) are the components of ψ_0 . The $(e^{-i\Lambda t})_{s's}$ are taken from (12) or (26).

The probability density for the whole process becomes

$$|\mathcal{A}^f|^2 = D_{s't'}^f (e^{-i\Lambda t})_{s's}^* (e^{-i\Lambda t})_{t't} \psi_s^*(0) \psi_t(0), \quad (50)$$

or, for an initial $K^0 (s = 1)$ or $\bar{K}^0 (s = -1)$, it becomes

$$R_s^f = D_{s't'}^f e_{s't',s}. \quad (51)$$

Here, the contributions from the kaon's time development $(e_{s't',s})$ and those from the decay process $(D_{s't'}^f)$ are neatly separated.

We have set

$$D_{s't'}^f = \mathcal{A}_{s't'}^{*f} \mathcal{A}_{t'}^f \quad (52)$$

$$e_{s't',s} = (e^{-i\Lambda t})_{s's}^* (e^{-i\Lambda t})_{t's}. \quad (53)$$

2.3.1 Semileptonic Decays

For the instant decay of a kaon to a final state ($\ell\pi\nu$) we define the four amplitudes

$$\mathcal{A}_{s'}^q = \langle \ell\pi\nu | \mathcal{H}_{\text{wk}} | s' \rangle, \quad q, s' = \pm 1, \quad (54)$$

with q : lepton charge (in units of positron charge), s' : strangeness of the decaying kaon.

We assume lepton universality. The amplitudes in (54) thus must not depend on whether ℓ is an electron or a muon.

Known physical laws impose constraints on these amplitudes:

The $\Delta S = \Delta Q$ rule allows only decays where the strangeness of the kaon equals the lepton charge, \mathcal{A}_q^q , and \mathcal{CPT} invariance requires (with lepton spins ignored) $\mathcal{A}_{-s'}^{-q} = \mathcal{A}_{s'}^{*q}$ [22]. The violation of these laws will be parameterized by the quantities x , \bar{x} , and $\text{Re}(y)$, posing

$$\mathcal{A}_{-1}^1 = x \mathcal{A}_1^1, \quad \mathcal{A}_1^{-1} = \bar{x}^* \mathcal{A}_{-1}^{-1}, \quad (|\mathcal{A}_1^1|^2 - |\mathcal{A}_{-1}^{-1}|^2) = -2\text{Re}(y) (|\mathcal{A}_1^1|^2 + |\mathcal{A}_{-1}^{-1}|^2),$$

or by $x_+ = (x + \bar{x})/2$ and $x_- = (x - \bar{x})/2$.

x_+ describes the violation of the $\Delta S = \Delta Q$ rule in \mathcal{CPT} -invariant amplitudes, x_- does so in \mathcal{CPT} -violating amplitudes.

The four probability densities for neutral kaons of strangeness $s = \pm 1$, born at $t = 0$, to decay at time t into $\ell\pi\nu$ with the lepton charge $q = \pm 1$ are, according to (51),

$$R_s^q = D_{s't'}^q e_{s't',s}, \quad \text{with} \quad D_{s't'}^q = \mathcal{A}_{s'}^{*q} \mathcal{A}_{t'}^q. \quad (55)$$

The decay rates, proportional to R_s^q , are given in [23].

We discuss here the asymmetries A_T and A_{CPT} with possible, additional symmetry breakings in the decay taken into account. The completed expressions are denoted by $A_T(t)$ and $A_{\text{CPT}}(t)$. Using Eqs. (55) and (12), we obtain

$$\begin{aligned} A_T(t) &= \frac{R_{-1}^1(t) - R_1^{-1}(t)}{R_{-1}^1(t) + R_1^{-1}(t)} \\ &= A_T - 2\text{Re}(y + x_-) + 2 \frac{\text{Re}(x_-)(e^{-\frac{1}{2}\Delta\Gamma t} - \cos(\Delta m t)) + \text{Im}(x_+) \sin(\Delta m t)}{\cosh(\frac{1}{2}\Delta\Gamma t) - \cos(\Delta m t)} \quad (56) \\ &\rightarrow A_T - 2\text{Re}(y + x_-) \text{ for } t \gg \tau_S, \quad (57) \end{aligned}$$

$$\begin{aligned} A_{\text{CPT}}(t) &= \frac{R_{-1}^{-1}(t) - R_1^1(t)}{R_{-1}^{-1}(t) + R_1^1(t)} \\ &= \frac{4\text{Re}(\delta + x_-/2) \sinh(\frac{1}{2}\Delta\Gamma\tau) + 4\text{Im}(\delta + x_+/2) \sin(\Delta m\tau)}{\cosh(\frac{1}{2}\Delta\Gamma\tau) + \cos(\Delta m\tau)} + 2\text{Re}(y) \quad (58) \\ &\rightarrow 4\text{Re}(\delta) + 2\text{Re}(y + x_-) \text{ for } t \gg \tau_S. \quad (59) \end{aligned}$$

We compare (58) with (47), and we recognize that, besides the additional term $\text{Re}(y)$, the new expression has the same functional behaviour, just with the old variable δ replaced by

$$\delta \rightarrow \delta + \frac{1}{2}(\text{Re}(x_-) + i \text{Im}(x_+)). \quad (60)$$

This shows that the analysis of a measurement, based on Eq. (58) alone, can not distinguish a possible \mathcal{CPT} violation in the time development from possible violations due to $\text{Re}(x_-)$ or $\text{Im}(x_+)$ in the decay.

We note that the combination

$$A_T(t \gg \tau_S) + A_{\text{CPT}}(t \gg \tau_S) = A_T + 4\text{Re}(\delta) \approx 4\text{Re}(\varepsilon) + 4\text{Re}(\delta) \quad (61)$$

yields a particular result on the kaon's time evolution that is free from symmetry violations in the decay !

Eqs. (56) to (60) are valid for $|x| \ll 1$, $|\bar{x}| \ll 1$, $|\text{Re}(y)| \ll 1$, and $|\delta| \ll 1$. For the last term in (61) we also assume $|\varepsilon| \ll 1$ and $\sigma \approx -1/2$.

Additional information on $\text{Re}(y + x_-)$ is gained by measuring the charge asymmetry δ_ℓ in the semi-leptonic decays of 'old kaons' K^0 (or \bar{K}^0) $\rightarrow \ell^\pm \pi \nu$.

For $t \gg \tau_S$ we obtain, up to first order in $|\varepsilon|$, $|\delta|$, $|y|$, and $|x_-|$, (and for $\sigma = -\frac{1}{2}$)

$$\delta_\ell = \frac{R_s^1 - R_s^{-1}}{R_s^1 + R_s^{-1}} = 2\text{Re}(\varepsilon - \delta) - 2\text{Re}(y + x_-) \quad (62)$$

independent of $s = 1$ or -1 .

An equivalent asymmetry A_S has been derived from the same rates integrated at short decay time [35,36]

$$A_S = 2\text{Re}(\varepsilon + \delta) - 2\text{Re}(y + x_-).$$

2.3.2 Decays to two pions - Decay rate asymmetries

The amplitude $\mathcal{A}^{\pi\pi}$ for the decay of the kaon $|\psi_0\rangle$ into two pions $|\pi\pi\rangle$ is, following (48),

$$\mathcal{A}^{\pi\pi} = \langle \pi\pi | \mathcal{H}_{\text{wk}} | s' \rangle (e^{-i\Lambda t})_{s's} \psi_s(0). \quad (63)$$

We express $|s'\rangle$ by the eigenstates $|K_\kappa\rangle$, $\kappa = \text{L or S}$, using (17) to (20), (22) and (23), and find $W^{\kappa s} = \langle \widetilde{K}_\kappa | s \rangle$ and

$$|s'\rangle (e^{-i\Lambda t})_{s's} = |K_\kappa\rangle e^{-i\lambda_\kappa t} \langle \widetilde{K}_\kappa | s \rangle, \quad (64)$$

and regain [26]

$$\mathcal{A}^{\pi\pi} = \mathcal{A}_\kappa^{\pi\pi} e^{-i\lambda_\kappa t} \langle \widetilde{K}_\kappa | s \rangle \psi_s(0) \quad (65)$$

where

$$\mathcal{A}_\kappa^{\pi\pi} = \langle \pi\pi | \mathcal{H}_{\text{wk}} | K_\kappa \rangle, \quad \kappa = \text{L or S}. \quad (66)$$

The decay rates are $\propto |\mathcal{A}^{\pi\pi}|^2$. For a K^0 at $t = 0$, we obtain

$$R_1^{\pi\pi} = \left| \mathcal{A}_\kappa^{\pi\pi} e^{-i\lambda_\kappa t} W^{\kappa 1} \right|^2 = \left| \mathcal{A}_S^{\pi\pi} e^{-i\lambda_S t} W^{11} + \mathcal{A}_L^{\pi\pi} e^{-i\lambda_L t} W^{21} \right|^2. \quad (67)$$

To calculate this expression it is convenient to use the following approximate eigenvectors, derived from Eqs. (37, 38), with $\sigma = -\frac{1}{2}$, and valid to first order in ε and δ ,

$$|K_S\rangle = N^S ((1 + \varepsilon + \delta) |K^0\rangle + (1 - (\varepsilon + \delta)) |\bar{K}^0\rangle), \quad (68)$$

$$|K_L\rangle = N^L ((1 + \varepsilon - \delta) |K^0\rangle - (1 - (\varepsilon - \delta)) |\bar{K}^0\rangle), \quad (69)$$

where $N^S = N^L \approx 1/\sqrt{2}$.

From Eqs. (21), (68), (69) we derive

$$W = (W^{ij}) \approx \frac{1}{\sqrt{2}} \begin{pmatrix} (1 - \varepsilon + \delta) & (1 + \varepsilon - \delta) \\ (1 - \varepsilon - \delta) & -(1 + \varepsilon + \delta) \end{pmatrix}, \quad (70)$$

and obtain, from (67), the rates of the decays $K^0 \rightarrow \pi\pi$ and $\bar{K}^0 \rightarrow \pi\pi$,

$$R_{\pm 1}^{\pi\pi}(t) = \frac{[1 \mp 2\text{Re}(\varepsilon - \delta)]}{2} \Gamma_S^{\pi\pi} \times [e^{-\Gamma_S t} + |\eta_f|^2 e^{-\Gamma_L t} \pm 2|\eta_f| e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t} \cos(\Delta m t - \phi_f)], \quad (71)$$

where $\Gamma_S^{\pi\pi}$ is the partial decay width of $K_S \rightarrow \pi\pi$, and where η_f equals

$$\eta_f = \frac{A_L^{\pi\pi}}{A_S^{\pi\pi}} = |\eta_f| e^{i\phi_f}. \quad (72)$$

In (71), terms of the order $|\varepsilon|^2$, $|\delta|^2$, and $|\eta_f| \delta$ are neglected.

The term $2\text{Re}(\varepsilon - \delta)$ may be of special use in some measurements, for example in the CPLEAR experiment, see Section 3.

A difference between the rates of the decays of K^0 and of \bar{K}^0 to the \mathcal{CP} eigenstates $\pi\pi$ is an indication of \mathcal{CP} violation.

For its study, the following rate asymmetries have been formed:

$$\begin{aligned} A_{\text{CP}}^f(t) &= \frac{R_{-1}^f(t) - R_1^f(t)}{R_{-1}^f(t) + R_1^f(t)} \\ &= 2\text{Re}(\varepsilon - \delta) - 2 \frac{|\eta_f| e^{\frac{1}{2}(\Gamma_S - \Gamma_L)t} \cos(\Delta m t - \phi_f)}{1 + |\eta_f|^2 e^{(\Gamma_S - \Gamma_L)t}}, \end{aligned} \quad (73)$$

where $f = \pi^+\pi^-$ (and $\eta_f = \eta_{+-}$) or $f = \pi^0\pi^0$ (and $\eta_f = \eta_{00}$).

2.3.3 Decays to two pions - Isospin analysis

The final states f may alternatively be represented by states with a definite total isospin I . The following three physical laws are expressed in terms of I , and can then be applied to the neutral kaon decay [37]: (i) the Bose symmetry of the two-pion states, (ii) the empirical $\Delta I = \frac{1}{2}$ rule, and (iii) the final state interaction theorem.

The Bose symmetry of the two-pion states forbids the pion pair to have $I = 1$. The $\Delta I = \frac{1}{2}$ rule in turn identifies the dominant transition K^0 (or \bar{K}^0) $\rightarrow |I = 0\rangle$. The final state interaction theorem, together with the assumption of $\mathcal{CP}\mathcal{T}$ invariance, relates the amplitudes $\langle I | \mathcal{H}_{\text{wk}} | K^0 \rangle$ and $\langle I | \mathcal{H}_{\text{wk}} | \bar{K}^0 \rangle$. It then naturally suggests a parametrization of $\mathcal{CP}\mathcal{T}$ violation in the decay process.

The relations

$$\langle \pi^+\pi^- | = \sqrt{2/3} \langle I = 0 | + \sqrt{1/3} \langle I = 2 | \quad (74)$$

$$\langle \pi^0\pi^0 | = \sqrt{1/3} \langle I = 0 | - \sqrt{2/3} \langle I = 2 | \quad (75)$$

transfer the implications of the laws mentioned to the observable final pion states.

We can now calculate the expressions of η_f in (72) for $\pi^+\pi^-$ and for $\pi^0\pi^0$, in terms of the decay amplitudes to the states with $I = 0, 2$.

If we denote $\langle I | \mathcal{H}_{\text{wk}} | K^0 \rangle = A_I e^{i\delta_I}$, then the final state interaction theorem asserts, that, if $\mathcal{CP}\mathcal{T}$ invariance holds, the corresponding amplitude for the antikaon decay is $\langle I | \mathcal{H}_{\text{wk}} | \bar{K}^0 \rangle = A_I^* e^{i\delta_I}$. δ_I is

the phase angle for $\pi\pi$ elastic scattering of the two pions at their center of momentum energy. Following [38] we violate $\mathcal{CP}T$ invariance by intruding the parameters B_I , $I = 0, 2$,

$$\langle I | \mathcal{H}_{\text{wk}} | K^0 \rangle = (A_I + B_I) e^{i\delta_I} \quad (76)$$

$$\langle I | \mathcal{H}_{\text{wk}} | \bar{K}^0 \rangle = (A_I^* - B_I^*) e^{i\delta_I}. \quad (77)$$

With the relations (74) to (77), and using (68), (69), we find the amplitudes $\langle \pi\pi | \mathcal{H}_{\text{wk}} | K_{L,S} \rangle$, and the observables η_{+-} and η_{00} [35]. We give them as follows

$$\eta_{+-} \approx \left[\varepsilon + i \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right] - \delta' + \varepsilon' \quad (78)$$

$$\eta_{00} \approx \eta_{+-} - 3\varepsilon' \quad (79)$$

with

$$\delta' = \left[\delta - \frac{\text{Re}(B_0)}{\text{Re}(A_0)} \right], \quad (80)$$

and with

$$\begin{aligned} \varepsilon' &\equiv e^{i\phi_{\varepsilon'}} \times |\varepsilon'| \\ &= e^{i\phi_{\varepsilon'}^{\text{CPT}}} \times \frac{1}{\sqrt{2}} \frac{\text{Re}(A_2)}{\text{Re}(A_0)} \left(\left[\frac{\text{Im}(A_2)}{\text{Re}(A_2)} - \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right] - i \left[\frac{\text{Re}(B_2)}{\text{Re}(A_2)} - \frac{\text{Re}(B_0)}{\text{Re}(A_0)} \right] \right), \end{aligned} \quad (81)$$

where

$$\phi_{\varepsilon'}^{\text{CPT}} = \left(\frac{\pi}{2} + \delta_2 - \delta_0 \right).$$

In (78) and (79), terms of second order in the \mathcal{CP} and $\mathcal{CP}T$ parameters, and of first order in ε' multiplied by $\text{Re}(A_2)/\text{Re}(A_0)$, are neglected [37].

2.3.4 Decays to two pions - With focus on $\mathcal{CP}T$ invariance

Equation (8b) relates the decay amplitudes A_0 and B_0 to the elements of Γ .

With approximating the decay rates by the dominating partial rates into the $\pi\pi$ states with $I = 0$, we have

$$\frac{\text{Im}(A_0)}{\text{Re}(A_0)} \approx -\frac{\text{Im} \Gamma_{K^0 \bar{K}^0}}{\Delta \Gamma} \quad (82)$$

$$\frac{\text{Re}(B_0)}{\text{Re}(A_0)} \approx \frac{\Gamma_{K^0 K^0} - \Gamma_{\bar{K}^0 \bar{K}^0}}{2\Delta \Gamma}, \quad (83)$$

and we recognize that

$$\delta' = |\delta'| e^{i(\phi_{\text{SW}} \pm \frac{\pi}{2})} = \text{Im}(\delta) (-\tan(\phi_{\text{SW}}) + i), \quad (84)$$

and

$$\frac{\text{Re}(B_0)}{\text{Re}(A_0)} = \delta - \delta' = \text{Re}(\delta) + \text{Im}(\delta) \times \tan(\phi_{\text{SW}}), \quad (85)$$

and that the terms $\left[\varepsilon + i \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right]$ and δ' in (78) are out of phase by $\pi/2$. See [35], as well as [23, 39], with [36] and [40], for a justification of the neglect of the other decay modes. (In Eq. (84) we had made use of $\text{Im}\delta' = \text{Im}\delta$).

Equation (85) relates the \mathcal{CPT} violating amplitude, of the dominating decay process into $\pi\pi$, with the \mathcal{CPT} violating parameter in the time evolution. Originally, these processes have been treated as independent.

ε' measures \mathcal{CP} violation in the decay process. From (81) we see that it is independent of the parameters of the time evolution. It is a sum of two terms. One of them is made exclusively of the decay amplitudes A_0 and A_2 , and is thus \mathcal{CPT} invariant. The other one contains the amplitudes B_0 and B_2 , and is thus \mathcal{CPT} violating. They are out of phase by $\frac{\pi}{2}$.

The value of the phase angle of the \mathcal{CPT} respecting part, $\phi_{\varepsilon'}^{\text{CPT}} = (\frac{\pi}{2} + \delta_2 - \delta_0)$, happens to be [41] roughly equal to ϕ_{+-}

$$\phi_{\varepsilon'}^{\text{CPT}} - \phi_{+-} \approx \text{few degrees} . \quad (86)$$

From the sine theorem, applied to the triangle of Eq. (79),

$$(\phi_{+-} - \phi_{00}) = \frac{3|\varepsilon'|}{|\eta_{00}|}(\phi_{\varepsilon'} - \phi_{+-}) , \quad (87)$$

we conclude that \mathcal{CPT} invariance in the decay process to two pions requires

$$|\phi_{+-} - \phi_{00}| \ll \text{few degrees} . \quad (88)$$

(We have used $|\phi_{\varepsilon'} - \phi_{+-}| \ll 1$ and $|\varepsilon'|/|\eta_{00}| \ll 1$).

On the other hand, the measured difference $\phi_{+-} - \phi_{00}$ limits the \mathcal{CPT} violating parameters in ε' as follows.

From (81) and (87) we obtain

$$\begin{aligned} \frac{\text{Re}(A_2)}{\text{Re}(A_0)} \left[\frac{\text{Re}(B_2)}{\text{Re}(A_2)} - \frac{\text{Re}(B_0)}{\text{Re}(A_0)} \right] &= -\text{Im}(\sqrt{2}|\varepsilon'| e^{i(\phi_{\varepsilon'} - \phi_{\varepsilon'}^{\text{CPT}})}) \\ &\approx \frac{\sqrt{2}}{3} |\eta_{00}|(\phi_{00} - \phi_{+-}) , \end{aligned} \quad (89)$$

and finally, with the use of the estimate $\frac{\text{Re}(A_2)}{\text{Re}(A_0)} \approx \left| \frac{A_2}{A_0} \right|$ (see [23, 42]), we arrive at

$$\frac{\text{Re}(B_2)}{\text{Re}(A_2)} = \frac{\text{Re}(B_0)}{\text{Re}(A_0)} + \frac{\sqrt{2}}{3} \left| \frac{A_0}{A_2} \right| |\eta_{00}|(\phi_{00} - \phi_{+-}) . \quad (90)$$

This equation, and (85), relate the \mathcal{CPT} violating expressions in ε' with the measured quantities.

For any two complex numbers η_{+-} and η_{00} with similar phase angles, $|\phi_{+-} - \phi_{00}| \ll 1$, we have to first order

$$\eta_{+-} \mp \eta_{00} = \{ |\eta_{+-}| \mp |\eta_{00}| \pm i|\eta_{00}|(\phi_{+-} - \phi_{00}) \} e^{i\phi_{+-}} . \quad (91)$$

If we allow for the approximation $e^{i(\phi_{+-} - \phi_{\varepsilon})} = 1$, we obtain from (79) and (91)

$$\text{Re}(\varepsilon'/\varepsilon) = \frac{1}{3} \left(1 - \frac{|\eta_{00}|}{|\eta_{+-}|} \right) \frac{|\eta_{+-}|}{|\varepsilon|} . \quad (92)$$

This quantity has been determined by a measurement of $|\eta_{00}/\eta_{+-}|^2$. See Section 3.

We apply (91) to

$$\eta \equiv \frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00} \quad (93)$$

and obtain (for $|\phi_{+-} - \phi_{00}| \ll 1$)

$$\eta = \left\{ \left(\frac{2}{3}|\eta_{+-}| + \frac{1}{3}|\eta_{00}| \right) - \frac{i}{3}|\eta_{00}|(\phi_{+-} - \phi_{00}) \right\} e^{i\phi_{+-}} = |\eta| e^{i\phi_\eta} \quad (94)$$

with

$$\phi_\eta = \frac{2}{3}\phi_{+-} + \frac{1}{3}\phi_{00} . \quad (95)$$

For the measured values see Section 4.

We eliminate now ε' from (78) and (79):

$$\varepsilon + i \frac{\text{Im}(A_0)}{\text{Re}(A_0)} - \delta + \frac{\text{Re}(B_0)}{\text{Re}(A_0)} = \eta , \quad (96)$$

and simplify this equation by setting the arbitrary phase angle ϑ in (10) to have

$$\Gamma_{K^0\bar{K}^0} = \text{real} , \quad (97)$$

making $\text{Im}(A_0)$ negligible

$$\text{Im}(A_0) \approx 0 . \quad (98)$$

This allows one, given $\text{Re}(\varepsilon) > 0$, to fix the phase angle of ε to ϕ_{SW}

$$\varepsilon = |\varepsilon| e^{i\phi_{\text{SW}}} \quad (99)$$

and to set $\sigma = -1/2$ (having neglected $|\varepsilon|^2 \ll 1$, $|\delta|^2 \ll 1$).

ε has now obtained the property to vanish, if \mathcal{T} invariance holds.

The term $\frac{\text{Re}(B_0)}{\text{Re}(A_0)}$ in (96) remains, as seen from (83), uninfluenced by the phase adjustment. We then obtain

$$\varepsilon - \delta + \frac{\text{Re}(B_0)}{\text{Re}(A_0)} = \eta . \quad (100)$$

Under \mathcal{CPT} invariance, this relation would be

$$\varepsilon = \eta = \frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00} , \quad (101)$$

with

$$\phi_\varepsilon \equiv \phi_{\text{SW}} = \phi_\eta \approx \phi_{+-} . \quad (102)$$

Applying (91) to $\varepsilon - \eta$ in (100) yields, with (39),

$$\begin{aligned} \delta_{\parallel} + i\delta_{\perp} &= \delta e^{-i\phi_{\text{SW}}} \\ &= |\varepsilon| - |\eta| + i|\eta|(\phi_{\text{SW}} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00}) + \frac{\text{Re}(B_0)}{\text{Re}(A_0)} e^{-i\phi_{\text{SW}}} , \end{aligned} \quad (103)$$

and with (40, 85)

$$\begin{aligned} M_{K^0\bar{K}^0} - M_{\bar{K}^0\bar{K}^0} &= 2|\Delta\lambda|\delta_{\perp} \\ &= 2|\Delta\lambda|\left\{ |\eta|(\phi_{\text{SW}} - \phi_\eta) - \frac{\text{Re}(B_0)}{\text{Re}(A_0)} \sin(\phi_{\text{SW}}) \right\} \\ &= 2|\Delta\lambda|\left\{ |\eta|(\phi_{\text{SW}} - \phi_\eta) - (\text{Re}(\delta) + \text{Im}(\delta)\tan(\phi_{\text{SW}})) \sin(\phi_{\text{SW}}) \right\}. \end{aligned} \quad (104)$$

All terms on the rhs are deduced from measurements.

\mathcal{CPT} invariance requires $M_{K^0 K^0} - M_{\bar{K}^0 \bar{K}^0} = \delta = 0$, and thus $(\phi_{\text{SW}} - \phi_\eta) = 0$. The comparison of the values of ϕ_{SW} and of $\phi_\eta \equiv \frac{2}{3}\phi_{+-} + \frac{1}{3}\phi_{00}$, done in Section 4, will confirm \mathcal{CPT} invariance.

Finally, combining (100) with the semileptonic charge asymmetry (62), we obtain

$$\text{Re}\left(\frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00}\right) - \frac{\delta_\ell}{2} = \frac{\text{Re}(B_0)}{\text{Re}(A_0)} + \text{Re}(y + x_-). \quad (105)$$

The terms on the rhs are \mathcal{CPT} violating.

2.3.5 Unitarity

The relation between the process of decay of the neutral kaon and the non-hermitian part Γ of Λ , expressed in the Eqs. (7) and (8b) offers the study of certain symmetry violations of \mathcal{H}_{wk} .

The terms in the sum for $\Gamma_{\alpha\alpha'}$ with $\alpha \neq \alpha'$, $\sum_\beta \langle \alpha | \mathcal{H}_{\text{wk}} | \beta \rangle \langle \beta | \mathcal{H}_{\text{wk}} | \alpha' \rangle$, express simultaneous transitions from different states $|\alpha'\rangle \neq |\alpha\rangle$ to one single final state $|\beta\rangle$. If the quantum numbers $\alpha' \neq \alpha$ represent conserved quantities, then the transitions to the single final state $|\beta\rangle$ would violate the conservation law in question.

Based on the fact that the occurrence of decay products requires a corresponding decrease of the probability of existence of the kaon, the following relation [43] holds

$$\text{Re}(\varepsilon) - i \text{Im}(\delta) = \frac{1}{2i\Delta m + \Gamma_S + \Gamma_L} \times \sum \langle f | \mathcal{H}_{\text{wk}} | K_L \rangle \langle f | \mathcal{H}_{\text{wk}} | K_S \rangle^*, \quad (106)$$

where the sum runs over all the final decay states f .

This equation has several remarkable aspects:

- (i) It is of great generality. Having admitted the time evolution to be of the general form (4), its validity is not restricted to perturbation theory or to \mathcal{CPT} invariance.
- (ii) The left-hand side (lhs) refers uniquely to the symmetry violations in the time evolution of the kaon, before decay, while the right-hand side (rhs) consists of the measurements, which include the complete processes.
- (iii) The rhs is dominated by the decays to $\pi^+\pi^-$ and $\pi^0\pi^0$. The other processes enter with the reduction factor $\Gamma_S/\Gamma_L \approx 580$, and, given their abundances, can often be neglected [23, 36, 40]. What remains of the sum, is approximately $\Gamma_S \eta_{\pi\pi}$, with $\eta_{\pi\pi}$ defined by

$$\eta_{\pi\pi} = |\eta_{\pi\pi}| e^{i\phi_{\pi\pi}} = \eta_{+-} \text{BR}_{\pi^+\pi^-}^S + \eta_{00} \text{BR}_{\pi^0\pi^0}^S \approx \frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00} \equiv \eta, \quad (107)$$

where the BR denote the appropriate branching ratios.

The measurements show $\eta_{\pi\pi} \approx \eta_{+-} \approx \eta_{00} \approx \eta$.

- (iv) The factor $1/(2i\Delta m + \Gamma_S + \Gamma_L)$ may be approximated by $\cos(\phi_{\text{SW}}) e^{-i\phi_{\text{SW}}}/\Gamma_S$, and thus

$$\text{Re}(\varepsilon) - i \text{Im}(\delta) \approx |\eta_{\pi\pi}| \cos(\phi_{\text{SW}}) e^{-i(\phi_{\text{SW}} - \phi_{\pi\pi})}. \quad (108)$$

Besides the results from the semileptonic decays, it is thus the phase in the decay to $\pi\pi$ which reveals the extent, to which the \mathcal{CP} violation in the time development, is a \mathcal{T} violation, and/ or a \mathcal{CPT} violation. Since the measurements yield $\phi_{\pi\pi} \approx \phi_{\text{SW}}$, the \mathcal{CP} violation is a \mathcal{T} violation with \mathcal{CPT} invariance.

We will later consider the hypothetical outcome $\phi_{\pi\pi} \approx \phi_{\text{SW}} + \pi/2$, which would signal a \mathcal{CP} violation with \mathcal{T} invariance and \mathcal{CPT} violation.

From Eq. (108) we note (since $|\phi_{\text{SW}} - \phi_{\pi\pi}| \ll 1$ and $\phi_{\pi\pi} \approx \phi_\eta$)

$$\text{Re}(\varepsilon) \approx |\eta_{\pi\pi}| \cos(\phi_{\text{SW}}) \quad (109)$$

$$\text{Im}(\delta) \approx |\eta_{\pi\pi}| \cos(\phi_{\text{SW}}) (\phi_{\text{SW}} - \phi_\eta) \approx \text{Re}(\varepsilon) (\phi_{\text{SW}} - \phi_\eta). \quad (110)$$

- (v) It is straight forward to formally recognize that the measured value of $\phi_{\pi\pi} \approx \phi_{\text{SW}}$ is in contradiction with $\mathcal{T}^{-1} \mathcal{H}_{\text{wk}} \mathcal{T} = \mathcal{H}_{\text{wk}}$. However, the experiment which measures $\phi_{\pi\pi}$ does not seem to involve any comparison of a process, running forward, with an identical one, but running backward.
- (vi) An analog mystery concerns \mathcal{CPT} invariance, as the measurement of $\phi_{\pi\pi}$ also does not obviously compare \mathcal{CPT} conjugated processes.
- (vii) Since the result of (106) is independent on possible symmetry violations in the decay, while the charge asymmetry δ_ℓ (62) contains such violations in the form of $\text{Re}(y + x_-)$, we may combine Eqs. (62) and (106) in view to evaluate this term.

Details of the application of (106) are found in [23, 40, 44, 45].

2.4 \mathcal{T} violation and \mathcal{CPT} invariance measured without assumptions on the decay processes

Some following chapters explain the measurements of $A_{\text{T}}(t)$ and of $A_{\text{CPT}}(t)$, performed in the CPLEAR experiment at CERN. These quantities are designed as comparisons of processes with initial and final states interchanged or, with particles replaced by antiparticles, and, as already shown above, they are intimately related to the symmetry properties of \mathcal{H}_{wk} . However, they include contributions from possible violations of symmetries in the decays, such as of \mathcal{CPT} invariance or of the $\Delta S = \Delta Q$ rule.

We will evaluate the sizes of $\text{Re}(y)$, $\text{Re}(y + x_-)$, and $\text{Im}(x_+)$, which constrain such violations to a negligible level.

As a preview, we recognize that the functions $A_{\text{T}}(t)$ and $A_{\text{CPT}}(t)$ consist of a part which is constant in time, and of a part which varies with time. The varying parts are rapidly decaying, and they become practically unimportant after $t \gtrsim 5\tau_S$. The two parts depend differently on the unknowns.

The constant parts already, of $A_{\text{T}}(t)$ and of $A_{\text{CPT}}(t)$, together with δ_ℓ , constitute three equations which show the feasibility to evaluate A_{T} , $\text{Re}(\delta)$, and $\text{Re}(y + x_-)$, and thus to determine an A_{T} , which is independent from assumptions on \mathcal{CPT} symmetry or from the $\Delta S = \Delta Q$ rule in the semileptonic decays.

This A_{T} depends uniquely on the time-reversal violation in the *evolution* of the kaon, and it is thus the direct measure for $\mathcal{T}^{-1} \mathcal{H}_{\text{wk}} \mathcal{T} \neq \mathcal{H}_{\text{wk}}$ searched for.

The $\text{Re}(\delta)$ in turn is a limitation of a hypothetical violation of $(\mathcal{CPT})^{-1} \mathcal{H}_{\text{wk}} (\mathcal{CPT}) = \mathcal{H}_{\text{wk}}$, also uniquely concerning the time evolution.

2.5 Time reversal invariance in the decay to $\pi\pi e^+ e^-$?

The decay of neutral kaons into $\pi^+ \pi^- \gamma$ has been studied in view of gaining information on symmetry violations which could not be obtained from the decay into $\pi^+ \pi^-$, especially on \mathcal{CP} violation of a different origin than the kaon's time evolution [46–49].

The existence of a sizable linear γ polarization and the possibility of its detection by internal pair conversion [50, 51], as well as the presence of a \mathcal{T} noninvariant term have been pointed out in [46].

Experiments have detected the corresponding \mathcal{T} - odd intensity modulation with respect to the angle between the planes of $(\pi^+ \pi^-)$ and $(e^+ e^-)$ in the decay $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ [52, 53]. As expected, the decay $K_S \rightarrow \pi^+ \pi^- e^+ e^-$ shows isotropy [53]. The data confirm a model [54], where, as usual, the \mathcal{CP} violation is also \mathcal{T} violating, and localized entirely in the time evolution of the kaon.

We discuss this result here, because its interpretation as a genuine example of a time-reversal noninvariance [54], or as a first direct observation of a time asymmetry in neutral kaon decay [55] has triggered critical comments [27, 30, 32, 56], with the concurring conclusion that, in the absence of final state interactions, the KTEV experiment at FNAL would find the same asymmetry when we assume there is no \mathcal{T} violation [32].

The enigma is explained in Ref. [56], whose authors remind us that a \mathcal{T} - odd term does not involve switching ‘in’ and ‘out’ states, and so is not a direct probe of \mathcal{T} violation.

As a complement, we wish to show that the model of [54] is an example, that a \mathcal{T} odd effect may well persist within \mathcal{T} invariance, even in the absence of final state interactions.

The γ radiation of $K_{L,S} \rightarrow \pi\pi\gamma$ has basically only two contributions, allowed by gauge invariance (up to third order in momenta) [49], which we refer to as E and as M. They have opposite space parity, and their space parity is opposite to their \mathcal{CP} parity. Since $\mathcal{CP}(\pi\pi) = +1$, we have $\mathcal{CP}(\pi\pi\gamma) = -\mathcal{P}(\gamma)$. In detail

$$\mathcal{CP}(\pi\pi\gamma) = -\mathcal{P}(\gamma) = \begin{cases} +1 & \text{E radiation} \\ -1 & \text{M radiation} \end{cases} \quad (111)$$

We thus see that the decays from the \mathcal{CP} eigenstates $K_1 \rightarrow \pi\pi\gamma_E$ and $K_2 \rightarrow \pi\pi\gamma_M$ are allowed within \mathcal{CP} invariance. A signal for \mathcal{CP} violation is (e. g.) the simultaneous occurrence of E and M radiation from a decaying old K^0 .

The variety of radiations is due to scalar factors, which multiply E and M, which are not determined by gauge invariance. They have to be measured or calculated from models.

The experiment [57] at CERN has identified the γ radiation from $K_S \rightarrow \pi\pi\gamma$ as pure low energy bremsstrahlung. This determines the scalar factor for the E radiation to be the one from soft photon emission, and fixes the phase of the $\pi\pi\gamma$ amplitude to be the one of the $\pi\pi$ amplitude [58]. This will become an important ingredient for the model [54] below.

The experiment [59] at BNL has found two similarly strong components in the γ radiation of $K_L \rightarrow \pi\pi\gamma$, (i) the bremsstrahlung, which is now \mathcal{CP} suppressed, and (ii) the M radiation, whose energy spectrum is compatible with a rise $\propto E_\gamma^3$. We can thus naturally expect that there is a value of the gamma ray energy E_γ , where the two components have equal intensity, and where thus the radiation shows a marked polarization due to interference.

The model of [54] calculates this polarization, and finds that the corresponding observable asymmetry in the distribution of the angle between the planes (e^+e^-) and ($\pi^+\pi^-$) is of the form

$$A \propto |\eta_{+-}| \sin(\phi_{+-} + \phi_{\text{FSI}}), \quad (112)$$

where ϕ_{FSI} is determined by the final state interaction theorem, and where ϕ_{+-} , as mentioned above, is fixed by the soft photon emission law.

In order for (112) to be a direct manifestation of \mathcal{T} violation, we would like to see A disappear, if \mathcal{T} violation is switched off, while \mathcal{CP} violation remains present.

Doing this, following [32] or (108), we see that A persists, if we set

$$|\eta_{+-}| \neq 0, \quad \phi_{+-} = \phi_{\text{SW}} + \frac{\pi}{2}, \quad \text{and} \quad \phi_{\text{FSI}} = 0.$$

The model presents thus a \mathcal{T} -odd observable which, in the absence of final state interactions, takes still a finite value when \mathcal{T} invariance holds.

2.6 Pure and mixed states

Until now we have implicitly assumed that a single neutral kaon represents a *pure* state, described by a state vector whose components develop in time coherently according to Eq. (4).

The *ensemble* of kaons in a beam is formed most often in individual reactions, and the kaons develop in time independently of each other. This ensemble represents a *mixed* state, and its description needs two state vectors and the knowledge of their relative intensity.

It is a deeply rooted property of quantum mechanics that the pure state of an isolated particle does not develop into a mixed state. Such a (hypothetical) transition would entail a loss of phase coherence of the amplitudes, and thus become detectable by the weakening of the interference patterns. It would also violate \mathcal{CPT} invariance [60,61], but in a different way than described in previous sections.

The various interference phenomena shown by neutral kaons have already been used as a sensitive detector in the search for coherence losses. As analysis tool in this search the density-matrix formalism used to describe mixed states seems appropriate.

2.6.1 Density matrix description

The time development of mixed states, and the results of measurements can be compactly described by the positive definite (hermitian) density matrix $\rho(t)$ [62–64].

All density matrices (in Quantum Mechanics, QM) develop in time in the same way, i. e. like those of pure states. A pure state ψ (with components $\psi_\kappa(t)$, $\kappa = K^0, \bar{K}^0$) has the density matrix $\rho(t) = \psi\psi^\dagger$ (with components $\psi_\kappa(t)\psi_{\kappa'}^*(t)$).

Density matrices thus develop according to $\rho(t) = e^{-i\Lambda t}\psi_0 (e^{-i\Lambda t}\psi_0)^\dagger$, or, denoting $U(t) = e^{-i\Lambda t}$ and $\rho(0) = \psi_0\psi_0^\dagger$, like

$$\rho(t) = U(t)\rho(0)U^\dagger(t), \quad t \geq 0, \quad (\text{QM}). \quad (113)$$

The form of (113) grants the conservation of

- (i) the *rank*, and
- (ii) the *positivity* of $\rho(t)$.

Since the pure states have (by construction) density matrices of *rank* = 1, the development (113) keeps pure states pure.

Since a matrix ρ of *rank* = 1 can always be written as a tensor product of two vectors, $\rho = \psi\psi^\dagger$, the developments by Eqs. (4) and (113) become equivalent for pure states.

Eq. (113) does not automatically conserve the trace, $\text{tr}\{\rho\}$, since $U(t)$ is not unitary. In order to avoid that the probability of existence of a neutral kaon does exceed the value one, we separately require, as a property of $U(t)$, that

$$1 \geq \text{tr}\{\rho(0)\} \geq \text{tr}\{\rho(t \geq 0)\}. \quad (114)$$

The outcome of measurements can be summarized as follows:

The probability W for a neutral kaon with the density matrix ρ , to be detected by an apparatus, tuned to be sensitive to neutral kaons with the density matrix ρ_f , is

$$W = \text{tr}\{\rho_f \rho\}. \quad (115)$$

2.6.2 Transitions from pure states to mixed states ?

It has been suggested [65] that gravitation might influence the coherence of wave functions and thereby create transitions from pure states to mixed states. These could look like a violation of Quantum Mechanics (QMV). In order to quantify observable effects due to such transitions, the authors of [66] have supplemented the Liouville equation of motion by a QM-violating term, linear in the density matrix. They have provided relations of the QMV parameters to a set of observables, to which the CPLEAR experiment has determined upper limits.

Our description includes extensions, specifications, and generalizations of the formalism.

In order to characterize the effects of QMV, it has been successful to introduce the Pauli matrices σ^μ , $\mu = 0, \dots, 3$ (with $\sigma^0 = \text{unit matrix}$) as a basis for the density matrices $\rho(t) \equiv \rho = R^\mu \sigma^\mu$, and $\rho(0) \equiv \rho_0 = R_0^\mu \sigma^\mu$. R^μ and R_0^μ are reals. We note that the determinant equals $|\rho| = R^\mu R_\mu$, and find from (113) that

$$R^\mu R_\mu = \|U\|^2 R_0^\mu R_{0\mu}. \quad (116)$$

$\|U\|$ is the absolute value of $\det(U)$. Indices are lowered with the 4×4 matrix $g = (g_{\mu\nu}) = (g^{\mu\nu})$, with $g^{00} = -g^{11} = -g^{22} = -g^{33} = 1$, $g^{\alpha\beta} = 0$ for $\alpha \neq \beta$.

Eq. (116) is a multiple of a *Lorentz* transformation [67] between the four-vectors $R \equiv (R^\mu)$ and $R_0 \equiv (R_0^\mu)$. We write its matrix as the exponential e^{Tt} , and the transformation as $R = e^{Tt}R_0$, where $T = (T^{\mu\nu}) = T^{00}\mathbf{1}_{4 \times 4} + L$, and where L is an element of the Lie algebra of the Lorentz transformations, and therefore satisfies

$$gLg = -L^T. \quad (117)$$

$()^T$ denotes the transpose of $()$.

Eq. (117) characterizes the quantum mechanical time evolution, which conserves the purity of the states, now expressed by $R^\mu R_\mu = 0$ (R light-like). An obvious way to let the formalism create transitions from pure states to mixed states is, to supplement the matrix T above with a matrix X , which is *not* an element of the Lie algebra of the Lorentz transformations, e. g. which satisfies

$$gXg = +X^T. \quad (118)$$

We will explicitly use

$$X = \begin{pmatrix} 0 & S^1 & S^2 & S^3 \\ -S^1 & -J^1 & D^3 & D^2 \\ -S^2 & D^3 & -J^2 & D^1 \\ -S^3 & D^2 & D^1 & -J^3 \end{pmatrix}. \quad (119)$$

The time evolution is now generated by the matrix

$$T = T^{00} \mathbf{1}_{4 \times 4} + L + X, \quad (120)$$

which is just a general 4×4 matrix. From (117) and (120) we see that QM has 7 parameters, and (119) shows that QMV has 9 parameters.

The probability for a neutral kaon, characterized by the four-vector R_0 at time $t = 0$, to be detected, by an apparatus set to be sensitive to R_f , is

$$W(t) = \text{tr}(\rho_f \rho(t)) = 2R_f^\mu (e^{Tt})^{\mu\nu} R_0^\nu \equiv 2 e^{T^{00}t} R_f^T e^{(L+X)t} R_0. \quad (121)$$

Using [64, 68]

$$e^{(L+X)t} = e^{Lt} e^{\mathbf{D}(t, -L, X)} = e^{\mathbf{D}(t, L, X)} e^{Lt} = \left(\mathbf{1} + \mathbf{D} + \dots + \frac{1}{n!} \mathbf{D}^n + \dots \right) e^{Lt} \quad (122)$$

with

$$\mathbf{D} \equiv \mathbf{D}(t, L, X) = \int_0^t d\tau e^{L\tau} X e^{-L\tau} = -\mathbf{D}(-t, -L, X), \quad (123)$$

we obtain to first order in X

$$W(t) = W_{QM}(t) + W_{QMV}^{(1)}(t) \quad (124)$$

$$W_{QM}(t) = 2 e^{T^{00}t} R_f^T e^{Lt} R_0 \quad (125)$$

$$W_{QMV}^{(1)}(t) = 2 e^{T^{00}t} R_f^T \mathbf{D} e^{Lt} R_0. \quad (126)$$

These equations have an evident interpretation: R_0 describes the kaon beam, R_f describes the detector, e^{Lt} describes the regular time evolution, and \mathbf{D} describes the decoherence. $W_{QM}(t)$ represents the result within QM.

In order to calculate the expressions (125), (126), we need T^{00} and L .

For $T = (T^{\mu\nu}) = T^{00} \mathbf{1}_{4 \times 4} + L$ we obtain, in terms of Λ ,

$$T^{\mu\nu} = \text{Im}(\text{tr}\{\sigma^\mu \Lambda \sigma^\nu\}), \quad (127)$$

and

$$T^{00} = -(\Gamma_S + \Gamma_L)/2, \quad (128)$$

and, to the lowest order,

$$L = \begin{pmatrix} 0 & -\Delta\Gamma/2 & 0 & 0 \\ -\Delta\Gamma/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta m \\ 0 & 0 & -\Delta m & 0 \end{pmatrix}, \quad (129)$$

and thus

$$e^{Lt} = \begin{pmatrix} \sigma^0 \cosh(\frac{\Delta\Gamma}{2}t) - \sigma^1 \sinh(\frac{\Delta\Gamma}{2}t) & (0) \\ (0) & \sigma^0 \cos(\Delta mt) + i\sigma^2 \sin(\Delta mt) \end{pmatrix}. \quad (130)$$

(130) will be used in Eqs. (122), (123) as the starting point for the calculation of $W(t)$, to any order in the QMV parameters in X , or in the small parameters in L . (We note in passing that $-L^{02} + iL^{13} = 2\epsilon \Delta\lambda = iL^{13}$ (having used $\phi_\epsilon = \phi_{SW}$), and that $L^{12} - iL^{03} = 2\delta \Delta\lambda$).

We are now able to list all possible experiments to search for QMV [67]. The four dimensions make R_0 and R_f capable to define four independent beams and four independent measurements, to give a total of 16 experiments.

It is a fortunate fact that \mathbf{D} in Eq. (126) introduces a sufficiently rich time dependence, which enables the existence of a specific set of four experiments [67, 69], that allows one to determine all 9 QMV parameters of (119).

As an example we study the influence of QMV on the decay of an initially pure K^0 into two pions. The expression for $R_{+1}^{\pi\pi}(t)$ in (71) corresponds to $W_{QM}(t)$. We calculate the modification $W_{QM}^{(1)}(t)$ due to QMV.

First, we verify that $R_f^T = \frac{1}{2}(1 \ 1 \ 0 \ 0)$ represents the state $\langle K_1 |$. We note that $\frac{1}{2}(1 \ 1 \ 0 \ 0) \hat{=} \frac{1}{2}(\sigma^0 + \sigma^1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \frac{1}{\sqrt{2}}(1 \ 1)$, and recognize that the last term is just the tensor product of $\psi_{K_1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$ with $\psi_{K_1}^\dagger$.

In the same way we find: $K_2 \hat{=} \frac{1}{2}(1 \ -1 \ 0 \ 0)$, $K^0 \hat{=} \frac{1}{2}(1 \ 0 \ 0 \ 1)$, and $\bar{K}^0 \hat{=} \frac{1}{2}(1 \ 0 \ 0 \ -1)$.

With $R_0 = \frac{1}{2}(1 \ 0 \ 0 \ 1)^T \hat{=} |K^0\rangle$ and $\mathbf{D} = J^1 \mathbf{D}(t, L, \frac{\partial}{\partial J^1} X)$, we obtain from (126)

$$W_{QM}^{(1)}(t) = \frac{J^1}{4\Delta\Gamma} e^{-\Gamma_L t} \quad \text{for } t \gg \tau_S. \quad (131)$$

Eqs. (124) to (126) together with (131) and (71) yield the modified expression for the decay rate

$$R_{+1}^{\pi\pi}{}_{QM}(t) \propto [a_{QM} e^{-\Gamma_S t} + |\eta_{QM}|^2 e^{-\Gamma_L t} + 2|\eta| e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t} \cos(\Delta mt - \phi)] \quad (132)$$

where

$$|\eta|^2 \rightarrow |\eta_{QM}|^2 = |\eta|^2 + \frac{J^1}{2\Delta\Gamma}. \quad (133)$$

The modification $1 \rightarrow a_{QM}$ of the short-lived term in (132) will not be considered further.

The outstanding features are the modification of $|\eta|^2$ in the long-lived term, in contrast to $|\eta|$ in the interference term, and the fact that the *first* order term of QMV, J^1 , combines with the *second* order term of the \mathcal{CP} violation, as seen in (133). This will allow one to determine an especially strict limit for J^1 [66, 70].

The parameters J^2 , J^3 , and D^1 are not present in (132). Contributions from further parameters in X are presently ignored, and discussed below.

Eq. (121), when $X \neq (0)$, does not guarantee positive values for $W(t)$, unless the values of the parameters of X fall into definite physical regions, since the time evolution generated by a general T of (120) does not satisfy (113).

We describe now the general law of time evolution of the density matrix and the ensuing physical region for the values in X .

The intriguing mathematical fact [71, 72] is, that Eq. (113) does guarantee the positivity of $W(t)$, not only

for the evolution of a single kaon, but also for a (suitably defined) system of many kaons. On the other hand, the precautions for the positivity (when $X \neq (0)$), tailored to the single-particle evolution, do, in general, *not* entail the positivity for the many-particle evolution, unless the single-particle evolution has the property of *complete positivity*. For the application to neutral kaons, see [69, 73, 74].

We summarize three results.

(i) Complete positivity is a necessary condition [69] for the consistent description of entangled neutral kaon pairs in a symmetric state, as produced in the $\bar{p}p$ annihilation [23, 75]. A general law of evolution therefore has to have this property.

The time evolution is completely positive if (and only if) it is given by (see [71])

$$\rho(t) = U_i(t)\rho(0)U_i^\dagger(t) , \quad (134)$$

where the right-hand side is a sum over four terms, with suitably normalized 2×2 matrices U_i [76].

(ii) The physical region for the values of the QMV parameters in X follows from (134) [69, 77]. The most important conditions are:

$$(D^i)^2 + (S^i)^2 \leq ((J^i)^2 - (J^j - J^k)^2)/4 , \quad (135)$$

$$J^i \geq 0 \quad \forall i = 1, \dots, 3 , \quad (136)$$

$$J^i \leq J^j + J^k , \quad (137)$$

(ijk = permutation of 123) .

We note:

If any one of the three diagonal elements, J^i , vanishes, then the other two ones are equal, and all off-diagonal elements, D^i and S^i , vanish, and
if any two of the three diagonal elements, J^i , vanish, then all elements of X vanish.

(iii) The condition $(1 \geq \text{tr}\{\rho(0)\} \geq \text{tr}\{\rho(t \geq 0)\})$ demands in addition

$$T^{00} \leq 0 , \quad (138)$$

$$(T^{00})^2 \geq (L^{01} + S^1)^2 + (L^{02} + S^2)^2 + (L^{03} + S^3)^2 . \quad (139)$$

This shows that, with the properties of neutral kaons, especially since $\Gamma_S \gg \Gamma_L$, there is little room for the values of the small parameters.

2.7 Entangled kaon pairs

Particles with a common origin may show a causal behaviour, still when they have become far apart, that is unfamiliar in a classical description.

Pairs of neutral kaons [78, 79] from the decay $\phi \rightarrow K^0 \bar{K}^0$, with the kaons flying away in opposite directions in the ϕ 's rest system, have a number of remarkable properties [26, 70]:

They are created in the entangled, antisymmetric $J^{PC} = 1^{--}$ state

$$|\psi_{-}\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle) \quad (140)$$

$$= \frac{1}{\sqrt{2}}(|K_2\rangle |K_1\rangle - |K_1\rangle |K_2\rangle) \quad (141)$$

$$\approx \frac{1}{\sqrt{2}}(|K_L\rangle |K_S\rangle - |K_S\rangle |K_L\rangle). \quad (142)$$

In each of these representations the two particles have opposite properties: opposite strangeness (140), opposite CP parity (141), or opposite shifts of the eigenvalues of Λ (142).

This allows the experimenter to define an ensemble of neutral kaons which has one of these properties with high purity [35, 80].

The identification of the particular quantum number shown by the particle which decays first, assures the opposite value for the surviving one.

This intriguing feature is not merely a consequence of conservation laws, since, at the moment of the pair's birth, there is nothing which determines, which particle is going to show what value of what observable, and when.

From (142), we see that $|\psi_{-}\rangle$ develops in time just by a multiplicative factor, $|\psi_{-}\rangle \rightarrow |\psi_{-}(t)\rangle = e^{-i(m_L+m_S)\gamma t} e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)\gamma t} |\psi_{-}\rangle$ which is independent of the symmetry violations ($\gamma t =$ eigentime of the kaons). $|\psi_{-}(t)\rangle$ and $|\psi_{-}\rangle$ have thus the same decay properties, e. g. the kaon pair cannot decay simultaneously into two $\pi\pi$ pairs or into the same $e\pi\nu$ triplets, at all times. It is due to this simplicity of the time evolution, and due to the antisymmetry of $|\psi_{-}\rangle$, that the kaons from ϕ decay are so well suited to explore symmetry violations in the decay processes, or to search for QMV, which ignores the states' antisymmetry.

For the formal description of a pair of neutral kaons in a general (mixed) state, we use the positive definite 4×4 density matrix $\rho(t_1, t_2)$. The times t_1, t_2 indicate the moments when later measurements on the individual particles will be performed. As a basis we use $(K^0\bar{K}^0, K^0K^0, \bar{K}^0K^0, \bar{K}^0\bar{K}^0)$, and we assume (with [26]), that $\rho(t_1, t_2)$ evolves like $\rho(t_1) \otimes \rho(t_2)$. The two-particle evolution is thus uniquely determined by the one-particle evolutions, and thus the introduction of QMV becomes obvious.

Similar to the one-particle 2×2 density matrices, we develop the 4×4 density matrix $\rho(t_1, t_2)$ in terms of the products $(\sigma^\mu \otimes \sigma^\nu)$ with coefficients $R^{\mu\nu} \equiv R^{\mu\nu}(t_1, t_2)$, $R_0^{\mu\nu} \equiv R^{\mu\nu}(0, 0)$, as

$$\rho(t_1, t_2) = R^{\mu\nu}(\sigma^\mu \otimes \sigma^\nu) \quad (143)$$

and obtain

$$R^{\mu\nu} = (e^{Tt_1})^{\mu\alpha} R_0^{\alpha\beta} (e^{Tt_2})^{\nu\beta}. \quad (144)$$

The generator T may, or may not, contain QMV terms.

The probability density that an apparatus, tuned to $\rho_f = R_f^{\mu\nu}(\sigma^\mu \otimes \sigma^\nu)$ detects the particles at the times t_1, t_2 is

$$\begin{aligned} W(t_1, t_2) &= \text{tr}(\rho_f \rho(t_1, t_2)) = 4R_f^{\mu\nu} R^{\mu\nu} = 4R_f^{\mu\nu} (e^{Tt_1})^{\mu\alpha} R_0^{\alpha\beta} (e^{Tt_2})^{\nu\beta} \\ &= 4R_f^{\mu\nu} (e^{(L+X)t_1})^{\mu\alpha} R_0^{\alpha\beta} (g e^{(-L+X)t_2} g)^{\beta\nu} e^{T^{00}(t_1+t_2)} \\ &\equiv 4 \text{Tr}\{ \mathbf{R}_f^T e^{(L+X)t_1} \mathbf{R}_0 g e^{(-L+X)t_2} g \} e^{T^{00}(t_1+t_2)}. \end{aligned} \quad (145)$$

The last expression in (145) is identical to the one before. The elements of the 4×4 matrices \mathbf{R}_0 and \mathbf{R}_f , e.g., are respectively, $R_0^{\alpha\beta}$ and $R_f^{\nu\mu}$. Tr acts on their superscripts.

Eq. (145) is the general expression for the measurements of all the parameters of the (general) $K^0\bar{K}^0$ pair.

In the deduction of (145), the equations (117) and (118) have been used. The different relative signs of L and X in the two exponents in (145) mark the difference of the effects due to QM or to QMV.

As a benefit of complete positivity, $W(t_1, t_2)$, for $t_1, t_2 \geq 0$, will be positive, if X satisfies the corresponding single-particle criteria.

We now give the general expression for the results of measurements on the pair (140). Its density matrix is

$$\begin{aligned} \rho(0,0)_- &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \frac{1}{\sqrt{2}} (0 \ 1 \ -1 \ 0) = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ &= \frac{1}{4} (\sigma^0 \otimes \sigma^0 - \sigma^m \otimes \sigma^m) = (g^{\mu\nu}/4) (\sigma^\mu \otimes \sigma^\nu), \end{aligned} \quad (146)$$

and thus

$$\mathbf{R}_0 = \mathbf{R}_0^- = g/4. \quad (147)$$

Inserting this interesting result, and (122), into (145), we obtain

$$W_{(K^0 \bar{K}^0)}(t_1, t_2) = \text{Tr} \{ \mathbf{R}_f^T e^{\mathbf{D}(t_1, L, X)} e^{L(t_1 - t_2)} e^{\mathbf{D}(t_2, L, X)} g \} e^{T^{00}(t_1 + t_2)}. \quad (148)$$

Again, all the terms in this expression, apart perhaps of g , have an obvious physical interpretation. Developing the exponentials $e^{\mathbf{D}}$, Eq. (148) allows one to calculate the frequency of occurrence of the events detected by the apparatus tuned to \mathbf{R}_f as a function of all the 16 parameters to any order in the small ones.

Explicit expressions have been published for 3 QMV parameters [70, 81], for 6 ones [77, 82], and for 9 ones [69].

Finally, we consider neutral-kaon pairs in the symmetric state

$$|\psi_+\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle |\bar{K}^0\rangle + |\bar{K}^0\rangle |K^0\rangle). \quad (149)$$

They have the density matrix [26]

$$\rho(0,0)_+ = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4} (\sigma^0 \otimes \sigma^0 + \sigma^m \otimes \sigma^m - 2 \sigma^3 \otimes \sigma^3), \quad (150)$$

and

$$\mathbf{R}_0 = \mathbf{R}_0^+ = \frac{1}{4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (151)$$

to be inserted into (145).

The explicit expression for $W(t_1, t_2)$ has been given, for the special case of QM, in [26].

$|\psi_+\rangle$, in contrast to $|\psi_-\rangle$, is allowed under QM, to evolve into $|K^0\rangle |K^0\rangle$ and into $|\bar{K}^0\rangle |\bar{K}^0\rangle$. \mathcal{CPT} forbids that $|\psi_+\rangle$ evolves into $\sim (|K_S\rangle |K_L\rangle + |K_L\rangle |K_S\rangle)$.

The treatment presented here is based on the description of the time evolution of the density matrix, generated by a general 4×4 matrix. An important difference to the regular quantum-mechanical time evolution is, that conservation laws do not follow anymore from symmetry properties, and that their existence is no more compulsory [66, 70, 81]. The question has to be left to the models, which enable QMV, whether the creator of QMV may also be the supplier of the otherwise missing conserved quantities.

3 Measuring neutral kaons

Many measurements concerning the neutral-kaon system have been carried out with beams containing a mixture of K^0 and \bar{K}^0 [44]. Neutral kaons are identified, among other neutral particles, by their masses, as obtained from measurements of their decay products.

The relative proportion of the numbers of K^0 and \bar{K}^0 particles at the instant of production is measured separately, and taken into account in the analysis. If the beam crosses matter, regeneration effects take place, and the measured ratio has to be corrected [83, 84].

Alternatively, K_S are separated from K_L taking advantage of the fact that a beam containing K^0 and \bar{K}^0 decays as a nearly pure K_S or K_L beam depending on whether it decays very near or far away from the source. This property was exploited at CERN in a precision measurement of the double ratio

$$\frac{\Gamma(K_L \rightarrow 2\pi^0)/\Gamma(K_L \rightarrow \pi^+\pi^-)}{\Gamma(K_S \rightarrow 2\pi^0)/\Gamma(K_S \rightarrow \pi^+\pi^-)} = |\eta_{00}/\eta_{+-}|^2 \approx 1 - 6\text{Re}(\varepsilon'/\varepsilon) \approx 1 - 6\varepsilon'/\varepsilon ,$$

which led to the discovery [85] of $\varepsilon' \neq 0$.

In a different approach neutral kaons have been identified and measured at their birth through the accompanying particles, in a convenient exclusive reaction. Conservation of energy and momentum allows neutral particles with the neutral-kaon mass to be selected. Differentiation between K^0 and \bar{K}^0 is achieved taking advantage of the conservation of strangeness in strong and electromagnetic interactions, through which kaons are produced. This dictates that the strangeness of the final state is equal to that of the initial state.

Thus, opposite-sign kaon beams have been used to produce K^0 and \bar{K}^0 by elastic charge-exchange in carbon, in order to compare K^0 and \bar{K}^0 decay rates to $\pi^+\pi^-$ [86].

Similarly, Ref. [87] reports measurements on $e\pi\nu$ decays from old K^0 obtained by inelastic charge-exchange of positive kaons in hydrogen.

CPLEAR [23] produced concurrently K^0 and \bar{K}^0 starting from $\bar{p}p$ annihilations, by selecting two charge-conjugate annihilation channels, $K^0K^-\pi^+$ and $\bar{K}^0K^+\pi^-$.

Another interesting $\bar{p}p$ annihilation channel is $K^0\bar{K}^0$, see [23]. The same state is exploited, as decay channel of the ϕ , at the ϕ factories, like the KLOE experiment does at DAΦNE [36]. Here, the speciality is, that neutral kaon pairs are created in entangled states. KLOE defines a neutral kaon as a K_S or a K_L according to the decay mode, or to the interaction, of the other neutral kaon of the pair.

In the course of the neutral-kaon time evolution, pionic and semileptonic decays may be used to define a fixed time t subsequent to the production time ($t = 0$).

Pionic ($\pi\pi$ and $\pi\pi\pi$) final states (which are \mathcal{CP} eigenstates or a known superposition of them) are suitable for \mathcal{CP} studies.

Semileptonic ($e\pi\nu$ and $\mu\pi\nu$) final states allow K^0 to be differentiated from \bar{K}^0 at the decay time, and are convenient for \mathcal{T} and \mathcal{CPT} studies.

Alternatively, in order to identify the strangeness at a time t , neutral kaons could be observed to interact in a thin slab of matter (in most cases bound nucleons), in a two-body reaction like $K^0p \rightarrow K^+n$ and $\bar{K}^0n \rightarrow K^-p$ or $\bar{K}^0n \rightarrow \pi^0\Lambda(\rightarrow \pi^-p)$, where the charged products reveal the strangeness of the neutral kaon.

As a case study we shall focus on the CPLEAR measurements, which yield results on \mathcal{T} violation and on \mathcal{CPT} invariance. Our presentation follows closely the description given by the CPLEAR group, summarized in Ref. [23].

3.1 CPLEAR experiment

Experimental method

Developing the ideas discussed in Ref. [88], CPLEAR chose to study the neutral-kaon time evolution by labelling (tagging) the state with its strangeness, at two subsequent times, see [23].

Initially-pure K^0 and \bar{K}^0 states were produced concurrently by antiproton annihilation at rest in a

hydrogen target, via the *golden channels*:

$$\bar{p}p \rightarrow \begin{matrix} K^- \pi^+ K^0 \\ K^+ \pi^- \bar{K}^0 \end{matrix}, \quad (152)$$

each having a branching ratio of $\approx 2 \times 10^{-3}$. The conservation of strangeness in the strong interaction dictates that a K^0 is accompanied by a K^- , and a \bar{K}^0 by a K^+ . Hence, the strangeness of the neutral kaon at production was tagged by measuring the charge sign of the accompanying charged kaon, and was therefore known event by event. The momentum of the produced $K^0(\bar{K}^0)$ was obtained from the measurement of the $\pi^\pm K^\mp$ pair kinematics. If the neutral kaon subsequently decayed to $e\pi\nu$, its strangeness could also be tagged at the decay time by the charge of the decay electron: in the limit that only transitions with $\Delta S = \Delta Q$ take place, neutral kaons decay to e^+ if the strangeness is positive at the decay time and to e^- if it is negative. This clearly was not possible for neutral-kaon decays to two or three pions.

For each initial strangeness, the number of neutral-kaon decays was measured as a function of the decay time t . These numbers were combined to form asymmetries – thus dealing mainly with ratios between measured quantities. However, the translation of measured numbers of events into decay rates requires (a) acceptance factors which do not cancel in the asymmetry, (b) residual background, and (c) regeneration effects to be taken into account. These experimental complications were handled essentially with the same procedure in the different asymmetries. Here we exemplify the procedure referring to $e\pi\nu$ decays.

- (a) Detecting and strangeness-tagging neutral kaons at production and decay relied on measuring, at the production (primary) vertex, a $K^\pm\pi^\mp$ track-pair and the corresponding momenta \vec{p}_{K^\pm} and \vec{p}_{π^\mp} , and, at the decay (secondary) vertex, an $e^\mp\pi^\pm$ track-pair and the corresponding momenta \vec{p}_{e^\mp} and \vec{p}_{π^\pm} . The detection (tagging) efficiencies of the $K^\pm\pi^\mp$ track-pairs depend on the pair charge configuration and momenta, and are denoted by $\epsilon(\vec{p}_{K^\pm}, \vec{p}_{\pi^\mp})$. A similar dependence exists for the detection efficiencies of the $e^\mp\pi^\pm$ track-pairs, $\epsilon(\vec{p}_{e^\mp}, \vec{p}_{\pi^\pm})$. Since the detection efficiencies of primary and secondary track-pairs were mostly uncorrelated, the acceptance of a signal ($e\pi\nu$) event was factorized as $\varrho_S \times \epsilon(\vec{p}_{K^\pm}, \vec{p}_{\pi^\mp}) \times \epsilon(\vec{p}_{e^\mp}, \vec{p}_{\pi^\pm})$. The factor ϱ_S represents the portion of the acceptance which does not depend on the charge configuration of either primary or secondary particles. The acceptances of the events corresponding to different charge configurations were then equalized (or normalized) by introducing two functions:

$$\xi(\vec{p}_K, \vec{p}_\pi) \equiv \frac{\epsilon(\vec{p}_{K^+}, \vec{p}_{\pi^-})}{\epsilon(\vec{p}_{K^-}, \vec{p}_{\pi^+})}, \quad (153a)$$

$$\eta(\vec{p}_e, \vec{p}_\pi) \equiv \frac{\epsilon(\vec{p}_{e^-}, \vec{p}_{\pi^+})}{\epsilon(\vec{p}_{e^+}, \vec{p}_{\pi^-})}. \quad (153b)$$

These functions, referred to as *primary-vertex normalization factor* and *secondary-vertex normalization factor*, respectively, are weights applied event by event, ξ to K^0 events and η to the events with a neutral kaon decaying to $e^+\pi^-$.

- (b) The background events mainly consist of neutral-kaon decays to final states other than the signal. Their number depends on the decay time t . To a high degree of accuracy the amount of background is the same for initial K^0 and \bar{K}^0 and hence cancels in the numerator but not in the denominator of any asymmetry: thus it is a dilution factor of the asymmetry. To account for these events, the analytic expressions of the asymmetries were modified by adding to the signal rates R and \bar{R} the corresponding background rates B and \bar{B} :

$$B(t) = \sum_i R_{Bi} \times \varrho_{Bi} / \varrho_S, \quad \bar{B}(t) = \sum_i \bar{R}_{Bi} \times \varrho_{Bi} / \varrho_S, \quad (154)$$

where R_{Bi} , \overline{R}_{Bi} are the rates of the background source i for initial K^0 and \overline{K}^0 , respectively, ϱ_S is defined above and ϱ_{Bi} is the corresponding term for the acceptance of events from the background source i . The quantities ϱ_{Bi} and ϱ_S were obtained by Monte Carlo simulation. Experimental asymmetries were formed from event rates including signal and background: $R^* = R + B$ and $\overline{R}^* = \overline{R} + \overline{B}$. These asymmetries were then fitted to the asymmetries of the *measured rates* (see below), which included residual background.

- (c) The regeneration probabilities of K^0 and \overline{K}^0 propagating through the detector material are not the same, thus making the measured ratio of initial \overline{K}^0 to K^0 decay events at time t different from that expected in vacuum [89]. A correction was performed by giving each K^0 (\overline{K}^0) event a weight w_r (\overline{w}_r) equal to the ratio of the decay probabilities for an initial K^0 (\overline{K}^0) propagating in vacuum and through the detector.

Finally, when $e\pi\nu$ decays were considered, each initial- K^0 event was given a total weight $w_+ = \xi \times \eta \times w_r$ or $w_- = \xi \times w_r$ if the final state was $e^+\pi^-\nu$ or $e^-\pi^+\overline{\nu}$, respectively. The summed weights in a decay-time bin are $N_{w+}(t)$ and $N_{w-}(t)$. In the same way, each initial- \overline{K}^0 event was given a total weight $\overline{w}_+ = \eta \times \overline{w}_r$ or $\overline{w}_- = \overline{w}_r$ if the final state was $e^+\pi^-\nu$ or $e^-\pi^+\overline{\nu}$. The corresponding summed weights are $\overline{N}_{w+}(t)$ and $\overline{N}_{w-}(t)$. In the case of decays to two or three pions, each initial- K^0 event was given a total weight $w = \xi \times w_r$, and each initial- \overline{K}^0 event a total weight $\overline{w} = \overline{w}_r$. The corresponding summed weights are $N_w(t)$ and $\overline{N}_w(t)$. In the following the summed weights are referred to as the *measured decay rates*. With these quantities are formed the *measured asymmetries*.

The measured asymmetries of interest here are

$$A_T^{\text{exp}}(t) = \frac{\overline{N}_{w+}(t) - N_{w-}(t)}{\overline{N}_{w+}(t) + N_{w-}(t)}, \quad (155a)$$

$$A_\delta^{\text{exp}}(t) = \frac{\overline{N}_{w+}(t) - \alpha N_{w-}(t)}{\overline{N}_{w+}(t) + \alpha N_{w-}(t)} + \frac{\overline{N}_{w-}(t) - \alpha N_{w+}(t)}{\overline{N}_{w-}(t) + \alpha N_{w+}(t)}, \quad (155b)$$

$$A_{+-}^{\text{exp}}(t) = \frac{\overline{N}_w(t) - \alpha N_w(t)}{\overline{N}_w(t) + \alpha N_w(t)}, \quad (155c)$$

The quantity $\alpha = 1 + 4\text{Re}(\varepsilon - \delta)$ is related to the primary vertex normalization procedure, see below. The phenomenological asymmetries to be fitted to each of the above expressions include background rates. Explicit expressions of the phenomenological asymmetries, in the limit of negligible background, can be written using (56), (58) and (73). For A_T^{exp} , as we shall see, Eq. (62) is also used.

(Two points are worth mentioning with regard to this method. Effects related to a possible violation of charge asymmetry in the reactions of Eq. (152) are taken into account by the weighting procedure at the primary vertex. When comparing the measured asymmetries with the phenomenological ones we take advantage of the fact that those reactions are strangeness conserving. A small strangeness violation (not expected at a level to be relevant in the CPLEAR experiment) would result in a dilution of the asymmetry and affect only some of the parameters.)

The detector

The layout of the CPLEAR experiment is shown in Fig. 1; a comprehensive description of the detector is given in Ref. [90].

The detector had a typical near- 4π geometry and was embedded in a (3.6 m long, 2 m diameter) warm solenoidal magnet with a 0.44 T uniform field (stable in a few parts in 10^4). The 200 MeV/c \overline{p} provided at CERN by the Low Energy Antiproton Ring (LEAR) [91] were stopped in a pressurized hydrogen gas target, at first a sphere of 7 cm radius at 16 bar pressure, later a 1.1 cm radius cylindrical target at 27 bar pressure.

A series of cylindrical tracking detectors provided information about the trajectories of charged particles. The spatial resolution $\sigma \approx 300 \mu\text{m}$ was sufficient to locate the annihilation vertex, as well as

the decay vertex if K^0 decays to charged particles, with a precision of a few millimetres in the transverse plane. Together with the momentum resolution $\sigma_p/p \approx 5$ to 10% this enabled a lifetime resolution of $\sigma \approx (5 - 10) \times 10^{-12}$ s.

The tracking detectors were followed by the particle identification detector (PID), which comprised a threshold Cherenkov detector, mainly effective for K/π separation above 350 MeV/c momentum ($> 4\sigma$), and scintillators which measured the energy loss (dE/dx) and the time of flight of charged particles. The PID was also used to separate e from π below 350 MeV/c.

The outermost detector was a lead/gas sampling calorimeter designed to detect the photons of the $K^0 \rightarrow 2\pi^0$ or $3\pi^0$ decays. It also provided e/π separation at higher momenta ($p > 300$ MeV/c). To cope with the branching ratio for reaction (3) and the high annihilation rate (1 MHz), a set of hardwired processors (HWP) was specially designed to provide full event reconstruction and selection in a few microseconds.

Selection of $e\pi\nu$ events

The $\bar{p}p$ annihilations followed by the decay of the neutral kaon into $e\pi\nu$ are first selected by topological criteria and by identifying one of the decay tracks as an electron or a positron, from a Neural Network algorithm containing the PID information. The electron spectrum and identification efficiency are shown in Fig. 2a.

The method of kinematic constrained fits was used to further reduce the background and also determine the neutral-kaon lifetime with an improved precision ($0.05 \tau_S$ and $0.2-0.3 \tau_S$ for short and long lifetime, respectively). The decay-time resolution was known to better than $\pm 10\%$. In total 1.3×10^6 events were selected, and one-half of these entered the A_T^{exp} asymmetry.

The residual background is shown in Fig. 2b. The simulation was controlled by relaxing some of the selection cuts to increase the background contribution by a large factor. Data and simulation agree well and a conservative estimate of 10% uncertainty was made. The background asymmetry arising from different probabilities of misidentifying π^+ and π^- , was determined to be 0.03 ± 0.01 by using $\bar{p}p$ multipion annihilations.

Each event selected, labeled by the initial kaon strangeness and the decay electron charge, was then properly weighted before forming the numbers N of events entering the asymmetries A_T^{exp} and A_δ^{exp} , see Eqs. (155a) and (155b).

Weighting $e\pi\nu$ events and building measured asymmetries

Regeneration was corrected on an event-by-event basis using the amplitudes measured by CPLEAR [92], depending on the momentum of the neutral kaon and on the positions of its production and decay vertices. Typically, this correction amounts to a positive shift of the asymmetry A_T^{exp} of 0.3×10^{-3} with an error dominated by the amplitude measurement.

The detection efficiencies common to the classes of events being compared in the asymmetries cancel; some differences in the geometrical acceptances are compensated to first order since data were taken with a frequently reversed magnetic field.

No cancellation takes place for the detection probabilities of the charged ($K\pi$) and ($e\pi$) pairs used for strangeness tagging, thus the two normalization factors ξ and η of Eqs. (153a) and (153b) were measured as a function of the kinematic configuration.

The factor ξ , which does not depend on the decay mode, was obtained from the data set of $\pi^+\pi^-$ decays between 1 and $4 \tau_S$, where the number of events is high and where the background is very small, see Ref. [93]. At any time t in this interval, after correcting for regeneration, and depending on the phase space configuration, the ratio between the numbers of decays of old K^0 and old \bar{K}^0 , weighted by ξ , is compared to the phenomenological ratio obtained from (73):

$$\frac{\xi N(K^0 \rightarrow \pi^+\pi^-)}{N(\bar{K}^0 \rightarrow \pi^+\pi^-)} = (1 - 4\text{Re}(\varepsilon - \delta)) \times \left(1 + 4|\eta_{+-}| \cos(\Delta m t - \phi_{+-}) e^{\frac{1}{2}\Gamma_S t} \right). \quad (156)$$

Thus, the product $\xi \times (1 + 4\text{Re}(\varepsilon - \delta))$ can be evaluated. The oscillating term on the right-hand side is known with a precision of $\approx 1 \times 10^{-4}$ (with the parameter values from Ref. [94]), and remains $< 4 \times 10^{-2}$. The statistical error resulting from the size of the $\pi^+\pi^-$ sample is $\pm 4.3 \times 10^{-4}$.

The effectiveness of the method is illustrated in Fig. 3. For the order of magnitude of ξ , as given by its average $\langle \xi \rangle$, CPLEAR quotes $\langle \xi \rangle = 1.12023 \pm 0.00043$, with $2\text{Re}(\varepsilon - \delta) \approx \delta_\ell = (3.27 \pm 0.12) \times 10^{-3}$ [94].

Some of the measured asymmetries formed by CPLEAR, (155b) and (155c), contain just the product $\xi \times [1 + 4\text{Re}(\varepsilon - \delta)]$, which is the quantity measured. However, for A_T^{exp} , (155b), ξ alone was needed. The analysis was then performed taking $\text{Re}(\varepsilon - \delta)$ from the measured K_L charge asymmetry, $\delta_\ell = 2\text{Re}(\varepsilon - \delta) - (\text{Re}(x_-) + \text{Re}(y))$. As a counterpart, the possible contribution to δ_ℓ of direct $\mathcal{CP}\mathcal{T}$ violating terms had to be taken into account.

The factor η was measured as a function of the pion momentum, using π^+ and π^- from $\bar{p}p$ multipion annihilations. The dependence on the electron momentum was determined using e^+e^- pairs from γ conversions, selected from decays $K^0(\bar{K}^0) \rightarrow 2\pi^0$, with a $\pi^0 \rightarrow 2\gamma$.

The value of η , averaged over the particle momenta, is $\langle \eta \rangle = 1.014 \pm 0.002$, with an error dominated by the number of events in the e^\pm sample.

The factors ξ and η are the weights applied event by event, which together with the regeneration weights, allowed CPLEAR to calculate the summed weights, in view of forming the measured asymmetries. The power of this procedure when comparing K^0 and \bar{K}^0 time evolution is illustrated in Figs. 4 and 5 for the $\pi^+\pi^-$ decay case.

The comparison of the measured asymmetries with their phenomenological expressions allows the extraction of the physics parameters, as reported in Section 4.

4 Measurements

4.1 $\mathcal{CP}\mathcal{T}$ invariance in the time evolution

What one measures is the parameter δ defined in (28a).

As for $\text{Re}(\delta)$, exploring the fact that $\text{Re}(y + x_-)$, which expresses $\mathcal{CP}\mathcal{T}$ violation in the semileptonic decay process, cancels out in the sum $A_T(t) + A_{\mathcal{CP}\mathcal{T}}(t)$, the CPLEAR group has formed a data set $A_\delta^{\text{exp}}(t)$ which measures this sum [95]. Using Eqs. (56), (58), and (45), for $A_{\mathcal{CP}\mathcal{T}}(t)$, $A_T(t)$ and A_δ , respectively, the measured quantity is shown to become

$$A_\delta^{\text{exp}}(t) = A_{\mathcal{CP}\mathcal{T}}(t) + A_T(t) - 4\text{Re}(\varepsilon - \delta) \quad (157)$$

$$= 8\text{Re}(\delta) + f(t, \text{Im}(x_+), \text{Re}(x_-), \text{Re}(\delta), \text{Im}(\delta)) . \quad (158)$$

The term $4\text{Re}(\varepsilon - \delta)$ follows from the normalization procedure, and the use of the decay rates to two pions (71). It does not require, however, a measurement of δ_ℓ (62).

The function f is given in [95]. It is negligible for $t \gtrsim 5\tau_S$.

Fig. 6 shows the data, together with the fitted curve $A_\delta^{\text{exp}}(t)$, calculated from the corresponding parameter values.

The main result is [95]

$$\text{Re}(\delta) = (0.30 \pm 0.33_{\text{stat}} \pm 0.06_{\text{syst}}) \times 10^{-3} .$$

The global analysis [45] gives a slightly smaller error

$$\text{Re}(\delta) = (0.24 \pm 0.28) \times 10^{-3} . \quad (159)$$

It confirms $\mathcal{CP}\mathcal{T}$ invariance in the kaon's time evolution, free of assumptions on the semileptonic decay process (such as $\mathcal{CP}\mathcal{T}$ invariance, or the $\Delta S = \Delta Q$ rule).

As for $\text{Im}(\delta)$, the most precise value

$$\text{Im}(\delta) = (0.000 \pm 0.020) \times 10^{-3}$$

is obtained by inserting $|\eta_{\pi\pi}| = (2.284 \pm 0.014) \times 10^{-3}$, $\phi_{\text{SW}} = (43.51 \pm 0.05)^\circ$, and $\phi_\eta = (43.5 \pm 0.7)^\circ$, all from [96], into (110).

A more detailed analysis [40] yields (within the statistical error) the same result.

The formula (110) also shows that the uncertainty of $\text{Im}(\delta)$ is, at present, just a multiple of the one of ϕ_η .

Using the Eqs. (32) and (39) with the values of $\text{Im}(\delta)$ given above, and of $\text{Re}(\delta)$ in (159), we obtain

$$\delta_{\parallel} = (0.17 \pm 0.20) \times 10^{-3}, \quad \delta_{\perp} = (-0.17 \pm 0.19) \times 10^{-3}.$$

The mass and decay-width differences then follow from Eqs. (40) and (41).

With $\Delta m = (3.48 \pm 0.01) \times 10^{-15}$ GeV and $\Delta\Gamma = (7.335 \pm 0.005) \times 10^{-15}$ GeV, we obtain

$$\begin{aligned} \Gamma_{K^0 K^0} - \Gamma_{\bar{K}^0 \bar{K}^0} &= (3.5 \pm 4.1) \times 10^{-18} \text{ GeV}, \\ M_{K^0 K^0} - M_{\bar{K}^0 \bar{K}^0} &= (-1.7 \pm 2.0) \times 10^{-18} \text{ GeV}. \end{aligned} \quad (160)$$

$\text{Im}(\delta)$ is constrained to a much smaller value than is $\text{Re}(\delta)$. $\text{Im}(\delta)$ could thus well be neglected. The results (160) are then, to a good approximation, just a multiple of $\text{Re}(\delta)$.

4.2 $\mathcal{CP}\mathcal{T}$ invariance in the semileptonic decay process

We can combine the result on $\text{Re}(\delta)$ with the measured values for δ_ℓ , Eq. (62), and $\text{Re}(\varepsilon)$, Eq. (109), and obtain

$$\text{Re}(y + x_-) = -\text{Re}(\delta) + \text{Re}(\varepsilon) - \delta_\ell/2. \quad (161)$$

For δ_ℓ the value

$$\delta_\ell = (3.27 \pm 0.12) \times 10^{-3} \quad (162)$$

can be used [97, 98].

For $\text{Re}(\varepsilon)$, when entering Eq. (109) with the values of $|\eta_{\pi\pi}|$ and with ϕ_{SW} given above, we have

$$\text{Re}(\varepsilon) = (1.656 \pm 0.010) \times 10^{-3}. \quad (163)$$

The value of $\text{Re}(y + x_-)$ thus obtained is in agreement with the one reached in a more sophisticated procedure by the CPLEAR group [45]:

$$\text{Re}(y + x_-) = (-0.2 \pm 0.3) \times 10^{-3}. \quad (164)$$

This result confirms the validity of $\mathcal{CP}\mathcal{T}$ invariance in the semileptonic decay process, as defined in 2.3.1. (The new, more precise values of δ_ℓ [99, 100] do not change this conclusion).

We note in passing [45]

$$\text{Re}(y) = (0.5 \pm 3.0) \times 10^{-3}.$$

4.3 \mathcal{T} violation in the kaon's time evolution

The measured asymmetry between the rates of $\bar{K}^0 \rightarrow e^+ \pi^- \nu$ and of $K^0 \rightarrow e^- \pi^+ \nu$ shall now be identified as an asymmetry between the rates of the mutually inverse processes $\bar{K}^0 \rightarrow K^0$ and $K^0 \rightarrow \bar{K}^0$, and thus be a demonstration of time reversal violation in the kaon's time evolution, revealing a violation of $\mathcal{T}^{-1} \mathcal{H}_{\text{wk}} \mathcal{T} = \mathcal{H}_{\text{wk}}$.

Based on Eq. (56), whose time independent part (57) becomes slightly modified by the normalization procedure [10], the CPLEAR data is expected to follow

$$A_{\text{T}}^{\text{exp}}(t) = A_{\text{T}} - 4\text{Re}(y + x_-) + g(t, \text{Re}(x_-), \text{Im}(x_+)). \quad (165)$$

The function g is given in [10]. It is negligible for $t \gtrsim 5\tau_S$.

Fig. 7 shows $\langle A_T^{\text{exp}} \rangle = 6.6 \times 10^{-3}$, $-4\text{Re}(y + x_-)$, and $g(t, \text{Re}(x_-), \text{Im}(x_+))$, calculated with the values for $\text{Re}(x_-)$ and $\text{Im}(x_+)$, given below, but increased by one standard deviation.

The main result is

$$\langle A_T^{\text{exp}} \rangle = (6.6 \pm 1.3_{\text{stat}} \pm 1.0_{\text{syst}}) \times 10^{-3},$$

in agreement with its theoretical prediction (45).

This is the only occasion in physics where a different transition rate of a subatomic process, with respect to its inverse, has been observed.

4.4 Symmetry in the semileptonic decay process ($\Delta S = \Delta Q$ rule)

The measurements discussed above allow one also to confirm the $\Delta S = \Delta Q$ rule for the semileptonic decay processes.

Much of the information is contained in the time dependent parts $f(t, \text{Im}(x_-), \text{Re}(x_-), \text{Re}(\delta), \text{Im}(\delta))$ and $g(t, \text{Re}(x_-), \text{Im}(x_+))$.

CPLEAR has found [23], [45]

$$\begin{aligned} \text{Re}(x_-) &= (-0.5 \pm 3.0_{\text{stat}} \pm 0.3_{\text{syst}}) \times 10^{-3}, \\ \text{Im}(x_+) &= (-2.0 \pm 2.6_{\text{stat}} \pm 0.5_{\text{syst}}) \times 10^{-3}. \end{aligned}$$

4.5 \mathcal{CPT} invariance in the decay process to two pions

A contribution to the study of \mathcal{CPT} invariance in the two-pion decay is the measurement [96]

$$\phi_{00} - \phi_{+-} = (0.2 \pm 0.4)^\circ,$$

which is in agreement with the request (88) that

$$|\phi_{+-} - \phi_{00}| \approx \left(\frac{1}{50}\right)^\circ.$$

The following \mathcal{CPT} -violating quantities have been given values, using Eqs. (85) and (90),

$$\frac{\text{Re}(B_0)}{\text{Re}(A_0)} = (0.24 \pm 0.28) \times 10^{-3}, \quad \frac{\text{Re}(B_2)}{\text{Re}(A_2)} = (0.32 \pm 0.32) \times 10^{-3}. \quad (166)$$

In addition to the above value for $(\phi_{+-} - \phi_{00})$, we have entered $\text{Re}(\delta)$ and $\text{Im}(\delta)$ from Section 4.1, $|A_2/A_0| = 0.0448 \pm 0.0002$ from [42], $|\eta_{00}|$ and ϕ_{SW} from [96]. See also [23].

All together \mathcal{CPT} invariance is confirmed. The measurements [96, 101] below fulfil (102):

$$\begin{aligned} \phi_{+-} &= (43.4 \pm 0.7)^\circ, \\ \phi_\eta = 2/3 \phi_{+-} + 1/3 \phi_{00} &= (43.5 \pm 0.7)^\circ, \\ \phi_{\text{SW}} &= (43.51 \pm 0.09)^\circ. \end{aligned}$$

In terms of the $K^0\bar{K}^0$ mass difference we note that all the terms on the rhs of (104) are negligible with respect to $\text{Re}(\delta)$ and we regain, in good approximation, Eq. (160)

$$M_{K^0K^0} - M_{\bar{K}^0\bar{K}^0} \approx -2|\Delta\lambda| \text{Re}(\delta) \sin(\phi_{\text{SW}}) = (-1.7 \pm 2.0) \times 10^{-18} \text{ GeV}.$$

Authors [40, 102] have considered the model, which assumes $\delta_{\parallel} = 0$ ($\hat{=} \Gamma_{K^0K^0} - \Gamma_{\bar{K}^0\bar{K}^0} = 0$), entailing $\delta = -\delta' = \text{Im}(\delta) (\tan(\phi_{\text{SW}}) - i)$, and thus leading to a roughly ten times more precise constraint. See also [23].

4.6 Further results on \mathcal{CPT} invariance

Each of the two terms on the rhs of Eq. (105), $\text{Re}(B_0)/\text{Re}(A_0)$ and $\text{Re}(y + x_-)$, vanishes under \mathcal{CPT} invariance. This is confirmed by the experimental results (164) and (166).

The experiment [99] has allowed one, in addition, to confirm the vanishing of their sum by the experimental determination of the lhs of Eq. (105)

$$\text{Re}\left(\frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00}\right) - \frac{\delta_\ell}{2} = (-0.003 \pm 0.035) \times 10^{-3} = \frac{\text{Re}(B_0)}{\text{Re}(A_0)} + \text{Re}(y + x_-). \quad (167)$$

Although these two terms represent two hypothetical \mathcal{CPT} violations of very unlike origins, we can see from Eqs. (85) and (161), that, with to-day's uncertainties on the values of $\text{Im}(\delta)$, $\text{Re}(\varepsilon)$, and δ_ℓ , their possible sizes are roughly equal to $\text{Re}(\delta)$:

$$\frac{\text{Re}(B_0)}{\text{Re}(A_0)} \approx -\text{Re}(y + x_-) \approx \text{Re}(\delta). \quad (168)$$

4.7 Transitions from pure states of neutral kaons to mixed states ?

The authors of [66] assume, for theoretical reasons, $J^3 = 0$. Taking complete positivity into account, we obtain $J^1 = J^2$, and all other elements of X vanish. This allows one to use (131) to (133). For $|\eta_{QMV}|$, the measurement of $R_s^{\pi\pi}(t)$ by [103], with the result $|\eta| = (2.30 \pm 0.035) \times 10^{-3}$, is well suited, since it would include effects of QMV. For $|\eta|$ in (133) we take the value of $|\varepsilon|$ reported in [12] from a first fit to CPLEAR data (mainly to the asymmetry $A_{\text{CP}}^f(t)$ of the decay rates to $f = \pi^+\pi^-$), with three of the QMV parameters left free. One could then evaluate that this result $|\varepsilon| = (2.34 \pm 0.08) \times 10^{-3}$ corresponds to the quantity $|\eta|$, which is free of QMV effects. Inserting the values above into (133), we obtain $\frac{1}{2}J^1 = (-1.4 \pm 2.9) \times 10^{-21}$ GeV. The analysis by [12] (for $\frac{1}{2}J^1 \equiv \gamma$, $\frac{1}{2}J^2 \equiv \alpha$, $\frac{1}{2}J^3 \equiv \beta$, all other QMV parameters = 0, and without the constraint of complete positivity) has yielded an upper limit (with 90 % CL) of

$$\frac{1}{2}J^1 < 3.7 \times 10^{-21} \text{ GeV}. \quad (169)$$

We note that this value is in the range of $\mathcal{O}(m_K^2/m_{\text{Planck}}) = 2 \times 10^{-20}$ GeV.

5 Conclusions

Measurements of interactions and decays of neutral kaons, which have been produced in well defined initial states, have provided new and detailed information on \mathcal{T} violation and on \mathcal{CPT} invariance in the time evolution and in the decay.

\mathcal{T} violation in the kaon evolution has been demonstrated by measuring that $\overline{K}^0 \rightarrow K^0$ is faster than $K^0 \rightarrow \overline{K}^0$, and by proving that this result is in straight conflict with the assumption, that \mathcal{T} and \mathcal{H}_{wk} would commute.

Complementary measurements have confirmed that hypothetical violations of the $\Delta S = \Delta Q$ rule or of \mathcal{CPT} invariance in the semileptonic decays, could not have simulated this result. See Fig. 7.

\mathcal{CPT} invariance is found intact. The combination of measurements on the decays to $\pi\pi$ and to $e\pi\nu$ yields constraints on parameters, which have to vanish under \mathcal{CPT} invariance, as well concerning the evolution as the decay processes.

The interplay of results from experiments at very high energies (CERN, FNAL) and at medium ones (CERN) has been displayed. A typical constraint on a hypothetical $K^0\overline{K}^0$ mass or decay width difference is a few times 10^{-18} GeV, resulting from the uncertainty of (the time evolution parameter) $\text{Re}(\delta)$.

In the future, more experiments with entangled neutral kaon-antikaon pairs, in an antisymmetric (140, 141, 142) or in a symmetric (149) state, will be performed.

The ϕ decay is a source of pairs in an antisymmetric state, which allows one to select a set of particles with precisely known properties. We wish to remind that pairs in the symmetric state have a complementary variety of phenomena, and also allow for a particular CPT test.

The experiments have achieved precisions which may open the capability to explore the validity of some of the often tacitly assumed hypotheses.

Some examples are: (i) the perturbation expansion of the Schrödinger equation [104, 105] leading to the two dimensional spinor representation with the exponential decay law, (ii) the unitarity relations [106], (iii) the conservation of the purity of states of isolated particles, manifested by the long time coherence of the kaon matter wave [65, 66].

A dedicated measurement on this subject has been performed at CERN. Data from the CPLEAR Collaboration in combination with earlier data from the CERN-HEIDELBERG Collaboration achieve a sensitivity of $\approx 10^{-21}$ GeV.

Comparisons of the long time coherence among kaons, neutrons and neutrini [107] are already possible.

Neutral kaons might well bring even more surprises. Probably the best probes in the world.

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Figure captions

Fig. 1 – CPLEAR detector.

(a) longitudinal view.

From the left, the 200 MeV/c \bar{p} beam delivered by LEAR enters the magnet along its axis, and through a thin scintillator (Beam monitor) reaches a pressurized hydrogen gas target (T) where antiprotons stop and annihilate. Cylindrical tracking detectors provide information about the trajectories of charged particles in order to determine their charge signs, momenta and positions: two proportional chambers (PC), six drift chambers (DC) and two layers of streamer tubes (ST). The particle identification detector (PID) comprising a threshold Cherenkov detector (C) sandwiched between two layers of scintillators (S1 and S2) allows the charged-kaon identification, and also the separation of electrons from pions. The outermost detector is a lead/gas sampling calorimeter (ECAL) to detect the photons from π^0 decays.

(b) transverse view and display of an event.

$\bar{p}p$ (not shown) $\rightarrow K^- \pi^+ K^0$ with the neutral kaon decaying to $e^- \pi^+ \bar{\nu}$. The view (b) is magnified twice with respect to (a) and does not show the magnet coils and outer detector components.

From Ref. [23].

Fig. 2 – Electron detection.

(a) Electron identification efficiency as a function of momentum when $< 2\%$ of pions fake electrons for real (\bullet) and simulated (\circ) calibration data;

(b) Decay time distribution for real (\blacksquare) and simulated (\diamond) data. The expected background contribution is shown by the solid line.

From Ref. [23].

Fig. 3 – \bar{K}^0 to K^0 normalization.

The ratio between the numbers of \bar{K}^0 and K^0 decays to $\pi^+ \pi^-$ in the $(1-4) \tau_S$ interval, \bar{N}_1/N_1^* (corrected for regeneration and \mathcal{CP} violating effects) as a function of the neutral-kaon transverse momentum p_T ,

(a) before and

(b) after giving each K^0 event its normalization weight.

The same ratio as a function of the decay vertex transverse separation from the production vertex d_T ,

(c) before and

(d) after applying the normalization weights.

From Ref. [93].

Fig. 4 – Neutral-kaon decay to $\pi^+ \pi^-$: the different decay rates indicate \mathcal{CP} violation.

The measured decay rates for K^0 (\circ) and \bar{K}^0 (\bullet) after acceptance correction and background subtraction.

From Ref. [93].

Fig. 5 – Neutral-kaon decay to $\pi^+ \pi^-$: the rate asymmetry $A_{+-}(t)$ demonstrates \mathcal{CP} violation.

The measured asymmetry $A_{+-}^{\text{exp}}(t)$, Eq. (155c), reduces to $A_{+-}(t)$ when the background is subtracted from the measured rates. The dots are the data points. The curve is the result of the fit making use of the rates (71).

From Ref. [23].

Fig. 6 – Experimental confirmation of \mathcal{CPT} invariance in the *time evolution* of neutral kaons.

The present result is the determinant input to a measurement of the decay-width and mass differences

between the neutral kaon and its antiparticle, the value of the latter being $(-1.7 \pm 2.0) \times 10^{-18}$ GeV . See Eqs. (160).

Values for times $t \gtrsim 5\tau_S$ depend entirely on a hypothetical \mathcal{CPT} violation in the time evolution, independently of further hypothetical violations of \mathcal{CPT} invariance in the *decay*, or of violations of the $\Delta S = \Delta Q$ rule.

The points are the measured values of A_δ^{exp} . The solid line represents the fitted curve (158). The dashes indicate a hypothetical violation of the $\Delta S = \Delta Q$ rule with the final CPLEAR values [45], exaggerated by one standard deviation for $\text{Im}(x_+)$ ($\rightarrow -4.6 \times 10^{-3}$), and for $\text{Re}(x_-)$ ($\rightarrow 2.5 \times 10^{-3}$).

The validity of the $\Delta S = \Delta Q$ rule is confirmed.

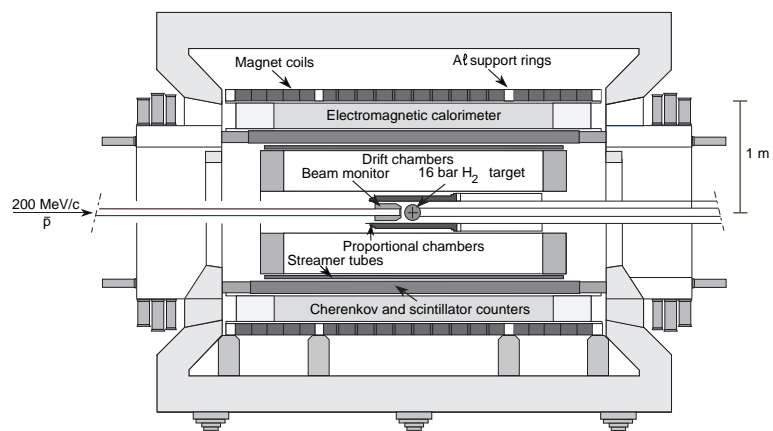
Data from Ref. [95]

Fig. 7 – Experimental demonstration of \mathcal{T} invariance violation in the *time evolution* of neutral kaons.

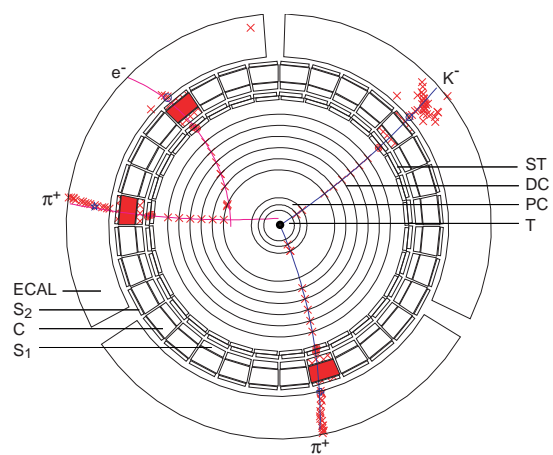
The positive values show that a \bar{K}^0 develops into a K^0 with higher probability than does a K^0 into a \bar{K}^0 . This *contradicts* $\mathcal{T}^{-1}\mathcal{H}_{\text{wk}}\mathcal{T} = \mathcal{H}_{\text{wk}}$ for neutral kaons.

The points are the measured values of $A_T^{\text{exp}}(t)$. The solid line represents the fitted constant $\langle A_T^{\text{exp}} \rangle$. All symmetry violating parameters concerning the semileptonic *decay process* are compatible with zero. The dashes indicate $-4\text{Re}(y + x_-) = 0.8 \times 10^{-3}$, the contribution of the constrained hypothetical \mathcal{CPT} invariance violation in the *decay*, expressed in Eq. (165). The time dependent curve, seen at early times, indicates a further constrained hypothetical violation of the $\Delta S = \Delta Q$ rule, expressed by the function g (with exaggerated input values).

Data from Ref. [10].



(a)



(b)

Figure 1:

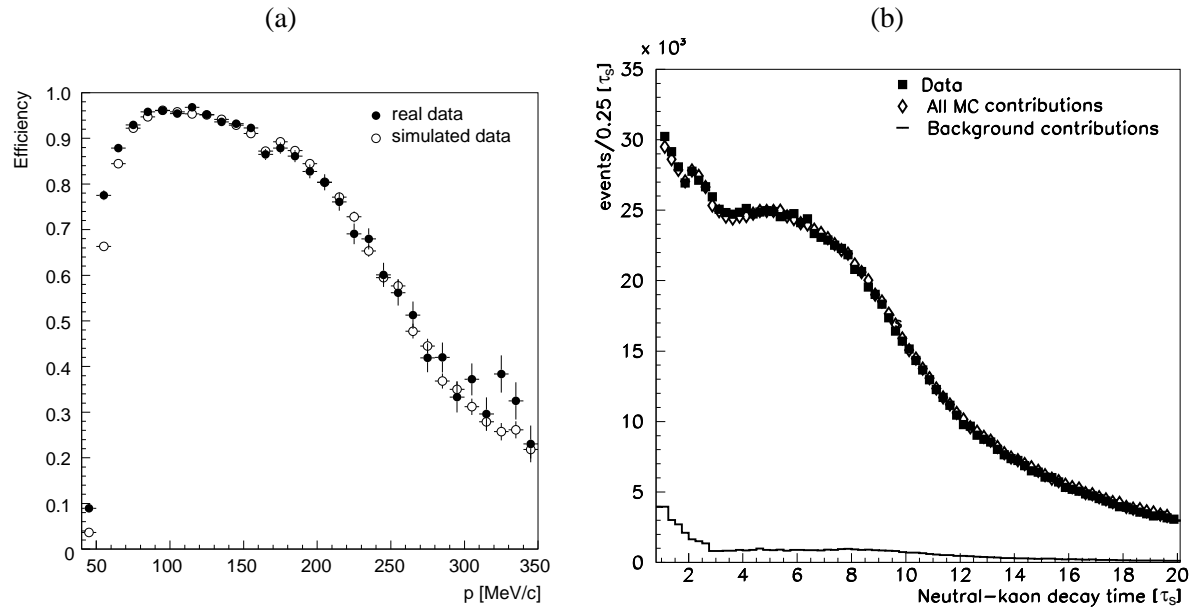


Figure 2:

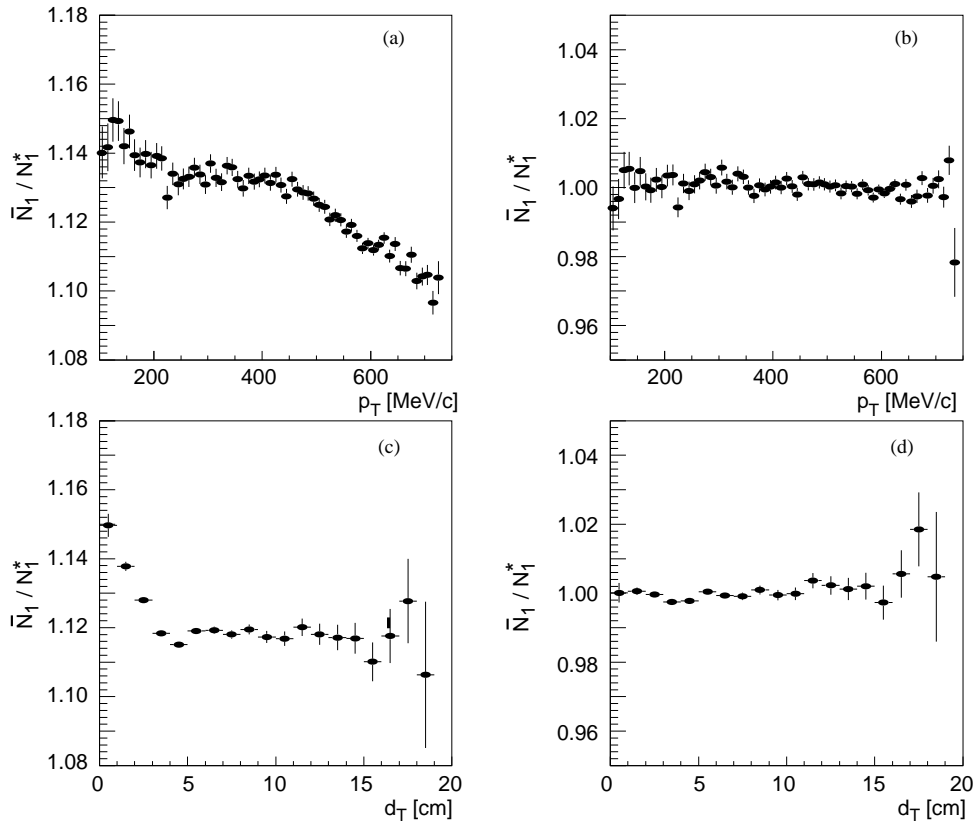


Figure 3:

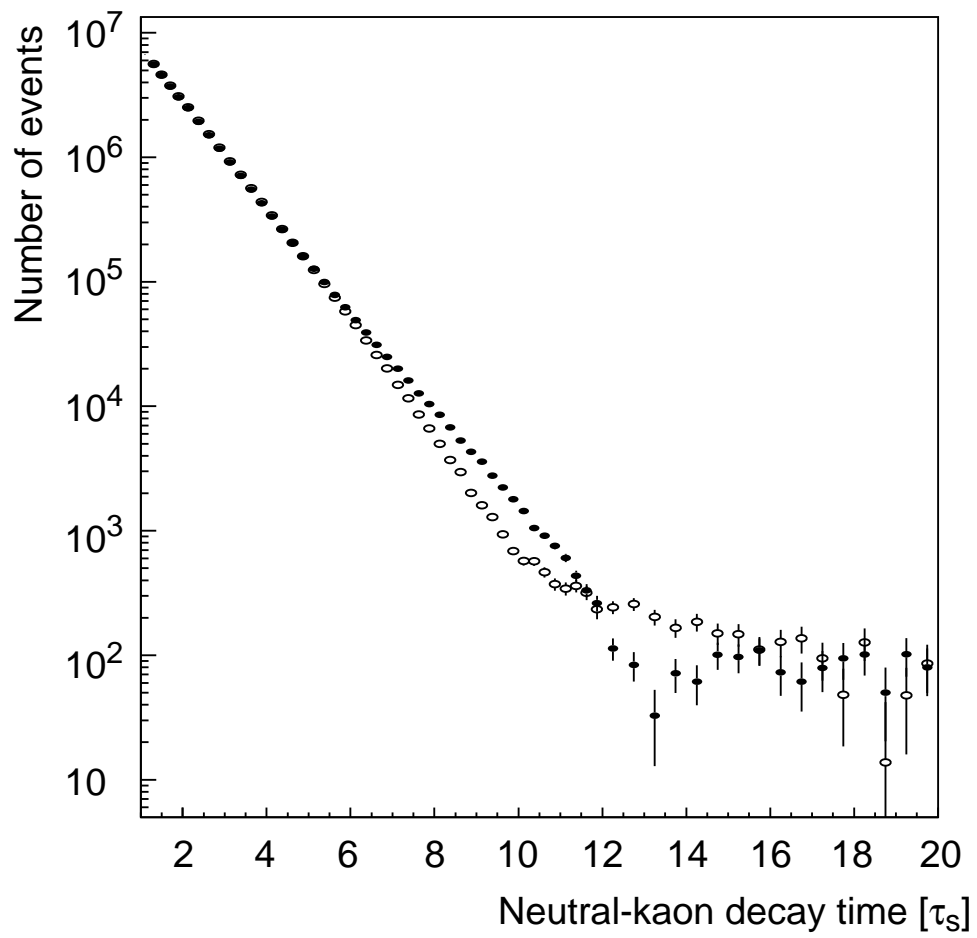


Figure 4:

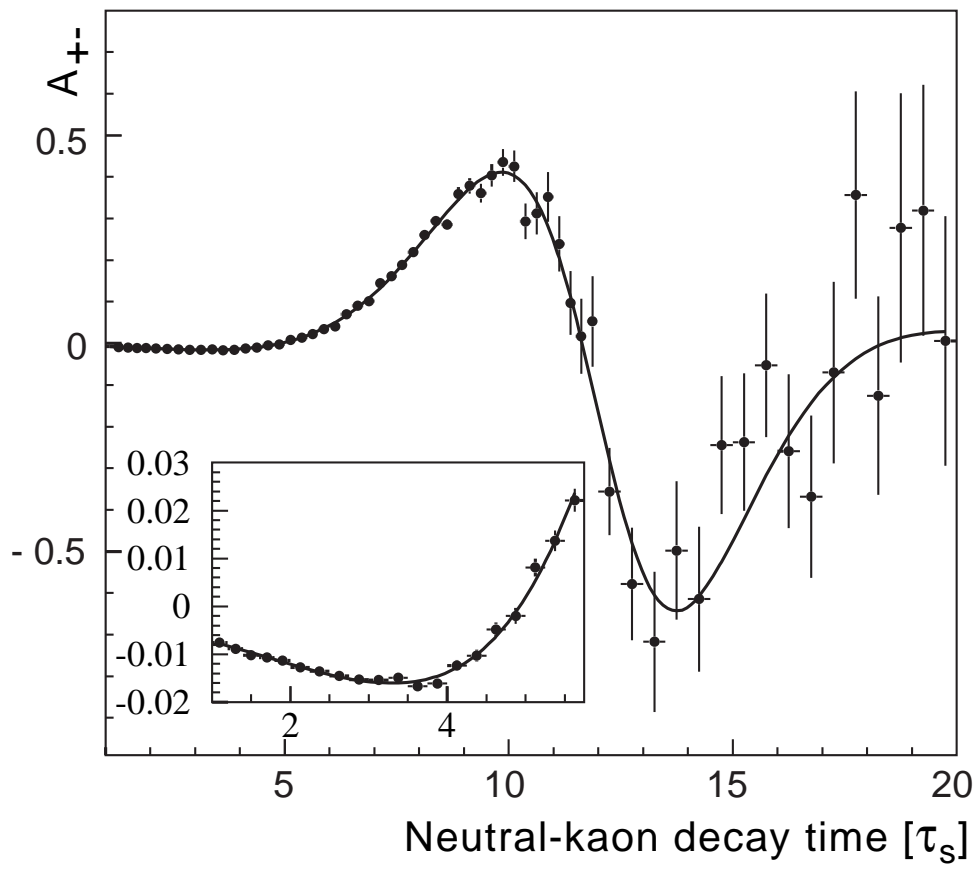


Figure 5:

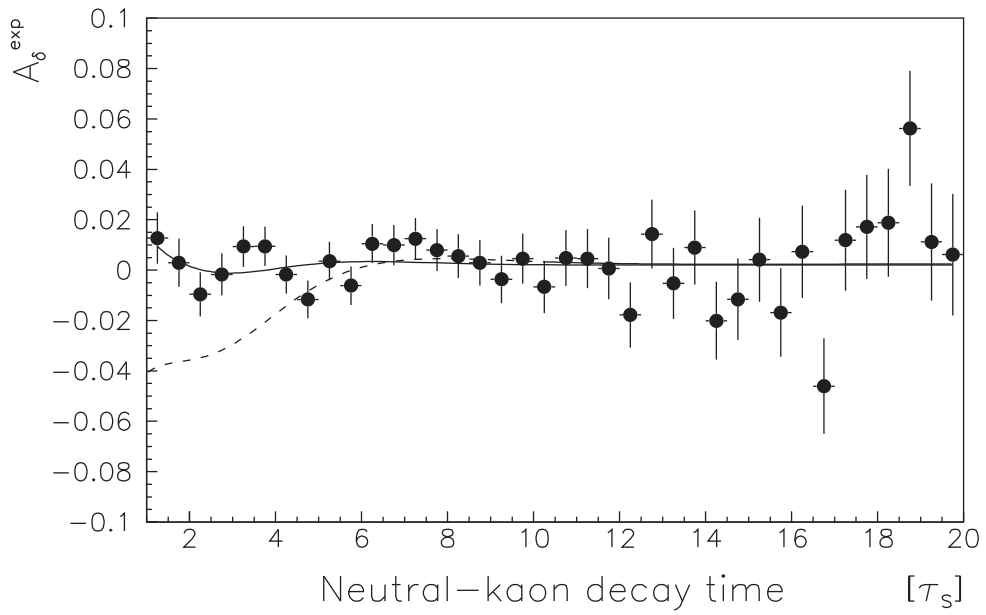


Figure 6:

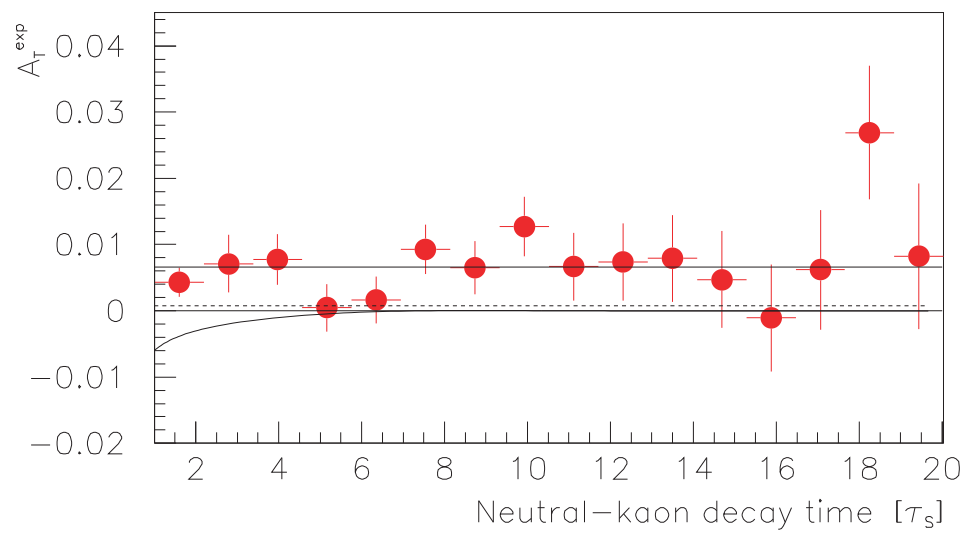


Figure 7:

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